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Rotating Black Hole in Extended Chern-Simons Modified Gravity

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We investigate a slowly rotating black hole in four-dimensional extended Chern-Simons modified gravity. We obtain an approximate solution that reduces to the Kerr solution when a coupling constant vanishes. The Chern-Simons correction effectively reduces the frame-dragging effect around a black hole in comparison with that of the Kerr solution.

Subject Index: 420, 453

Introduction There is growing interest in modifications of the theory of general relativity. The theory of quantum gravity mostly motivates such modifications.[1),2)] Quantum gravity is requisite for dealing with extreme situations in the Universe, such as the vicinity of a black hole and the initial state of the Universe. For the theory of quantum gravity, several candidates have been proposed.[1),2)] Those candidates certainly require modifications of general relativity. In recent years, serious astrophysical problems of dark matter and dark energy also motivate modified gravity theories. According to recent observational results,[3)] the dark components are considered to comprise over 90% of the energy constituents in the Universe. Modifications of general relativity would provide an alternative way to resolve the unsolved issues of the dark components. Among modified gravity theories, we focus on four-dimensional Chern-Simons (CS) modified gravity[4),5)] in this paper.

In CS modified gravity, the action is modified by adding a CS term to the Einstein-Hilbert action. Deser et al.[6)] originally developed the CS modified gravity in (2+1) dimensions and Jackiw and Pi[4)] extended it to (3+1) dimensions by introducing an external scalar function. Furthermore, Smith et al.[5)] have recently considered an extended CS modified gravity in which the scalar function is regarded as a dynamical scalar field. The most interesting points of the four-dimensional CS modified gravity[4),5),7)–13)] are summarized as follows. First, the CS modified gravity can be obtained explicitly from the superstring theory.[5),7)] In the superstring theory, a CS term in the Lagrangian density is essential to cancel anomaly. Second, as shown by Weinberg,[14)] the parity-violating higher-order correction in an effective field theory for inflation also provides the CS modified gravity. Third, the Schwarzschild solution holds in the CS modified gravity.[4)] Thus, the theory passes the classical tests of general relativity. Finally, in the CS modified gravity, the axial part of gravitational fields is mainly modified in comparison with the cases of gen-

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eral relativity. In fact, the Kerr solution does not hold in the CS modified gravity. This is because the Chern-Pontryagin density, which can be derived from the partial integral of the CS term, is closely coupled to the axial part of gravitational fields. In relation to the last point, rotating black hole solutions have been discussed in the framework of the Jackiw-Pi model.[8)–10)] In Ref. 8), an approximate solution for a rotating black hole was obtained by using a slow rotation approximation. The approximate solution interestingly shows the same feature as that of flat rotation curves seen in spiral galaxies.[9)] In Ref. 10), an exact solution for a rotating mathematical black hole was obtained. Somewhat exotic features of those solutions are considered to arise from the constraint of vanishing Chern-Pontryagin density in the Jackiw-Pi model. In this paper, we investigate an approximate solution for a rotating black hole in the extended CS modified gravity, in which the constraint is replaced with a field equation for a scalar field. To obtain a black hole solution, we assume both slow rotation of a black hole and a weak CS coupling in this study. Throughout the paper, we use geometrized units with $c = G = 1$.

Extended CS modified gravity The extended CS modified gravity is provided by the action[4),5)]

$$I = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi}R + \frac{\ell}{64\pi}\vartheta\, {}^*R^{\tau}{}_{\sigma}{}^{\mu\nu}R^{\sigma}{}_{\tau\mu\nu} - \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\vartheta\right)\left(\partial_\nu\vartheta\right) + \mathcal{L}_{\rm m}\right], \quad (1)$$

where g is the determinant of the metric $g_{\mu\nu}$, $R \equiv g^{\alpha\beta}R_{\alpha\beta}$ ($R_{\alpha\beta} \equiv R^{\lambda}{}_{\alpha\lambda\beta}$) is the Ricci scalar, $R^{\tau}{}_{\sigma\alpha\beta} \equiv \partial_\beta \Gamma^{\tau}_{\sigma\alpha} - \cdots$ is the Riemann tensor ($\Gamma^{\alpha}_{\beta\gamma}$ is the Christoffel symbols), ℓ is a coupling constant, ϑ is a dynamical scalar field, and $\mathcal{L}_{\rm m}$ is the Lagrangian for matter. The dual Riemann tensor is defined by ${}^*R^{\tau}{}_{\sigma}{}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}R^{\tau}{}_{\sigma\alpha\beta}$, where $\varepsilon^{0123} \equiv 1/\sqrt{-g}$ is the Levi-Civita tensor. In this paper, we neglect the surface integral term (see Ref. 11) for a detailed discussion) and the potential term for ϑ in the action. When we neglect the kinematic term of ϑ in Eq. (1), the action reduces to the Jackiw-Pi model developed in Ref. 4). From the variations in the action with respect to the metric $g_{\mu\nu}$ and the scalar field ϑ, we obtain the field equations, respectively,

$$G^{\mu\nu} + \ell C^{\mu\nu} = -8\pi\left({T_{\rm m}}^{\mu\nu} + {T_\vartheta}^{\mu\nu}\right), \quad (2)$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\vartheta = -\frac{\ell}{64\pi}\, {}^*R^{\tau}{}_{\sigma}{}^{\mu\nu}R^{\sigma}{}_{\tau\mu\nu}, \quad (3)$$

where $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is the Einstein tensor, $C^{\mu\nu}$ is the Cotton tensor defined by

$$C^{\mu\nu} \equiv -\frac{1}{2}\left[\left(\nabla_\sigma\vartheta\right)\left(\varepsilon^{\sigma\mu\alpha\beta}\nabla_\alpha R^{\nu}{}_{\beta} + \varepsilon^{\sigma\nu\alpha\beta}\nabla_\alpha R^{\mu}{}_{\beta}\right) + \left(\nabla_\sigma\nabla_\tau\vartheta\right)\left({}^*R^{\tau\mu\sigma\nu} + {}^*R^{\tau\nu\sigma\mu}\right)\right], \quad (4)$$

${T_{\rm m}}^{\mu\nu}$ is the energy-momentum tensor for matter, and ${T_\vartheta}^{\mu\nu}$ is the energy-momentum tensor of the scalar field ϑ,

$${T_\vartheta}^{\mu\nu} = \left(\nabla^\mu\vartheta\right)\left(\nabla^\nu\vartheta\right) - \frac{1}{2}g^{\mu\nu}(\nabla^\lambda\vartheta)(\nabla_\lambda\vartheta). \quad (5)$$

Thus, Eqs. (2) and (3) are basic equations for gravitational fields under the extended CS modified gravity.

Approximate solution for a rotating black hole We discuss a slowly rotating black hole by considering the perturbation of a spherically symmetric static spacetime. For such a background, we have the metric $ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$ and the scalar field $\vartheta = \vartheta^{(0)}(r)$, where Φ and Λ are functions of r, and the superscript '(0)' denotes the zeroth order in perturbation. The field equations for the background reduce to[12)]

$$G^{\mu\nu}_{(0)} = -8\pi T_{\vartheta^{(0)}}{}^{\mu\nu}, \tag{6}$$

$$g^{\mu\nu}_{(0)} \nabla^{(0)}_\mu \nabla^{(0)}_\nu \vartheta^{(0)} = 0. \tag{7}$$

Hereafter, we assume ${T_{\rm m}}^{\mu\nu} = 0$. We now adopt the Schwarzschild spacetime with $\vartheta^{(0)} = 0$ as a background solution for Eqs. (6) and (7). The metric in polar coordinates is given by

$$g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{8}$$

where M is the mass of a black hole. Let us consider the perturbation of the Schwarzschild spacetime,

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}, \tag{9}$$

$$\vartheta = \vartheta^{(1)}, \tag{10}$$

where $g^{(1)}_{\mu\nu}$ and $\vartheta^{(1)}$ are at least first-order quantities of a small parameter ϵ that characterizes the slow rotation of a black hole, i.e., $g^{(1)}_{\mu\nu}, \vartheta^{(1)} \sim O(\epsilon)$. Thus, we take account of a scalar field that is purely excited by the spacetime curvature through Eq. (3). As discussed in detail below, we also adopt the approximation of a weak CS coupling, that is, we also regard ℓ as an expansion parameter. The scalar field obeys Eq. (3). While the right-hand side in Eq. (3), i.e., ${}^*{R^\tau}_\sigma{}^{\mu\nu} {R^\sigma}_{\tau\mu\nu}$, exactly vanishes for spherically symmetric spacetimes, ${}^*{R^\tau}_\sigma{}^{\mu\nu} {R^\sigma}_{\tau\mu\nu}$ becomes nonzero at first order in ϵ. Thus, the scalar field should be of order $\ell\epsilon$. Then the CS correction in Eq. (2) has order $\ell^2\epsilon$. Therefore, when we take the Schwarzschild black hole as a background, the CS correction to the rotation of the black hole is of order $\ell^2\epsilon$.

We discuss the basic equations for such a slowly rotating black hole in the extended CS modified gravity. Using the Regge-Wheeler gauge,[15)] we have the first-order axisymmetric metric as

$$g^{(1)}_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2M}{r}\right) h(r,\theta) & H(r,\theta) & 0 & -r^2\sin^2\theta\omega(r,\theta) \\ H(r,\theta) & \left(1 - \frac{2M}{r}\right)^{-1} m(r,\theta) & 0 & -r^2 u(r,\theta) \\ 0 & 0 & r^2 k(r,\theta) & 0 \\ -r^2\sin^2\theta\omega(r,\theta) & -r^2 u(r,\theta) & 0 & r^2 k(r,\theta)\sin^2\theta \end{pmatrix}, \tag{11}$$

where $(h, H, m, k, \omega, u) \sim O(\epsilon)$. The scalar field is also assumed to depend on r and θ, i.e., $\vartheta = \vartheta^{(1)}(r,\theta) \sim O(\epsilon)$. We do not pay attention to the order in ℓ for a while.

We can obtain differential equations for the first-order functions $(h, H, m, k, \omega, u, \vartheta)$ from Eqs. (2) and (3). Differential equations of first order in ϵ are completely divided into four groups as $\{h, m, k\}$, $\{H\}$, $\{u\}$, and $\{\omega, \vartheta\}$. The (tt), (rr), $(r\theta)$, $(\theta\theta)$, and $(\phi\phi)$ components of Eq. (2) give homogeneous differential equations for h, m, and k. The (tr) and $(t\theta)$ components of Eq. (2) give homogeneous differential equations for H, and the $(r\phi)$ and $(\theta\phi)$ components give homogeneous differential equations for u. For those components of Eq. (2), we have $C^{\mu\nu} \sim O\left(\epsilon^2\right)$. Of course, a simple solution of $h = m = k = H = u = 0$ satisfies the differential equations for h, m, k, H, and u. Finally, we obtain differential equations for ω and ϑ from the $(t\phi)$ component of Eq. (2) and the field equation (3) for the scalar field,

$$\begin{aligned} & r(r-2M)\partial_r^2\omega + 4(r-2M)\partial_r\omega + \partial_\theta^2\omega + 3\cot\theta\partial_\theta\omega \\ & \quad = \ell\frac{6M(r-2M)}{r^4}\left(\frac{1}{\sin\theta}\partial_r\partial_\theta\vartheta - \frac{1}{r\sin\theta}\partial_\theta\vartheta\right), \end{aligned} \tag{12}$$

$$\begin{aligned} & r(r-2M)\partial_r^2\vartheta + 2(r-M)\partial_r\vartheta + \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\vartheta\right) \\ & \quad = -\ell\frac{3M}{8\pi r}\left(\sin\theta\partial_r\partial_\theta\omega + 2\cos\theta\partial_r\omega\right). \end{aligned} \tag{13}$$

Equations (12) and (13) govern the rotation of a black hole under the extended CS modified gravity.

We discuss solutions for Eqs. (12) and (13) to clarify how the black hole rotates. For this purpose, let us expand ω and ϑ as

$$\omega(r,\theta) = \sum_{n=1}^{\infty}\tilde{\omega}_n(r)\frac{1}{\sin\theta}\partial_\theta P_n(\cos\theta), \tag{14}$$

$$\vartheta(r,\theta) = \sum_{n=1}^{\infty}\tilde{\vartheta}_n(r)P_n(\cos\theta), \tag{15}$$

where P_n denotes the Legendre function of the first kind. From Eqs. (12) and (13), we obtain for each n

$$\tilde{\omega}_n'' + \frac{4}{r}\tilde{\omega}_n' + \frac{2-n(n+1)}{r(r-2M)}\tilde{\omega}_n = \ell\frac{6M}{r^5}\left(\tilde{\vartheta}_n' - \frac{\tilde{\vartheta}_n}{r}\right), \tag{16}$$

$$\tilde{\vartheta}_n'' + \frac{2(r-M)}{r(r-2M)}\tilde{\vartheta}_n' - \frac{n(n+1)}{r(r-2M)}\tilde{\vartheta}_n = \ell\frac{3n(n+1)M}{8\pi r^2(r-2M)}\tilde{\omega}_n', \tag{17}$$

where the prime denotes the differentiation with respect to r. To solve Eqs. (16) and (17), let us assume $\ell \ll 1$. We solve Eqs. (16) and (17) by the method of iteration by considering ℓ to be an expansion parameter. The homogeneous differential equations of Eqs. (16) and (17) give the equations of order $\ell^0\epsilon$. We obtain the homogeneous solutions as

$$\tilde{\omega}_n = A_0\,{}_2F_1\left(1-n, 2+n, 4; \frac{r}{2M}\right) + B_0\,G_{2,2}^{2,0}\left(\frac{r}{2M}\middle|\begin{array}{ccc} -n-1 & , & n \\ -3 & , & 0 \end{array}\right), \tag{18}$$

$$\tilde{\vartheta}_n = C_0\,P_n\left(\frac{r}{M}-1\right) + D_0\,Q_n\left(\frac{r}{M}-1\right), \tag{19}$$

where A_0, B_0, C_0, and D_0 are constants, ${}_2F_1$ is a hypergeometric function, $G^{2,0}_{2,2}$ is a Meijer G-function,[16] and Q_n is the Legendre function of the second kind. Note that while $P_n\left(r/M-1\right)$ diverges at $r\to\infty$, $Q_n\left(r/M-1\right)$ diverges at $r=2M$. Thus, we should have $C_0=D_0=0$ at order $\ell^0\epsilon$. Otherwise, $\tilde{\omega}_n$ would also diverge at $r=2M$ or $r\to\infty$ at higher orders. The metric solution in Eq. (18) for $n=1$ gives the linear approximation of the Kerr solution. The solution for $n=1$ allows us to compare a rotating black hole in the extended CS modified gravity with that in general relativity. Hence, we focus on the solution for $n=1$ hereafter. The solution of order $\ell^0\epsilon$ is summarized as

$$\tilde{\omega}_1=-\frac{2J}{r^3},\quad \tilde{\vartheta}_1=0, \tag{20}$$

where J is the angular momentum of a black hole. Substituting Eq. (20) into the right-hand side of Eq. (17), we obtain a solution of order $\ell\epsilon$,

$$\begin{aligned}\tilde{\vartheta}_1=&C_1\,P_1\left(\frac{r}{M}-1\right)+D_1\,Q_1\left(\frac{r}{M}-1\right)\\&-\ell\frac{J}{256\pi M^5r^4}\Bigg[2M\left(15r^4+5M^2r^2+10M^3r+18M^4\right)\\&-15r^4(r-M)\ln\left|\frac{r}{r-2M}\right|\Bigg],\end{aligned} \tag{21}$$

where C_1 and D_1 are constants. When we impose the regularity of $\tilde{\vartheta}_1$ both at $r=2M$ and at $r\to\infty$, we derive

$$C_1=0,\quad D_1=-\ell\frac{15J}{256\pi M^5}. \tag{22}$$

Thus, we obtain the regular solution of order $\ell\epsilon$ as

$$\tilde{\vartheta}_1=-\ell\frac{J}{128\pi M^2r^4}\left(5r^2+10Mr+18M^2\right). \tag{23}$$

In a similar way, substituting this solution into the right-hand side of Eq. (16), we obtain the solution of order $\ell^2\epsilon$ as

$$\tilde{\omega}_1=\ell^2\frac{J}{1792\pi Mr^8}\left(70r^2+120Mr+189M^2\right). \tag{24}$$

Consequently, we obtain the approximate solution up to order $\ell^2\epsilon$ for a slowly rotating black hole,

$$\omega=\frac{2J}{r^3}\left[1-\ell^2\frac{1}{3584\pi Mr^5}\left(70r^2+120Mr+189M^2\right)\right], \tag{25}$$

$$\vartheta=-\ell\frac{J}{128\pi M^2r^4}\left(5r^2+10Mr+18M^2\right)\cos\theta. \tag{26}$$

From Eq. (25), we obtain the metric solution for a slowly rotating black hole under the extended CS modified gravity

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2$$

$$-\frac{4J}{r}\left[1-\ell^2\frac{1}{3584\pi Mr^5}\left(70r^2+120Mr+189M^2\right)\right]\sin^2\theta dtd\phi. \tag{27}$$

At infinity, the $(t\phi)$ component of the metric and scalar field reduce to

$$g_{t\phi} \simeq -\frac{2J}{r}\left(1-\ell^2\frac{5}{256\pi Mr^3}\right)\sin^2\theta, \tag{28}$$

$$\vartheta \simeq -\ell\frac{5J}{128\pi M^2r^2}\cos\theta. \tag{29}$$

At $r \simeq 2M$, we have

$$g_{t\phi} \simeq -\left[\frac{J}{M}\left(1-\ell^2\frac{709}{114688\pi M^4}\right)-\frac{J}{2M^2}\left(1-\ell^2\frac{1727}{57344\pi M^4}\right)(r-2M)\right]\sin^2\theta, \tag{30}$$

$$\vartheta \simeq -\ell J\left[\frac{29}{1024\pi M^4}-\frac{43(r-2M)}{1024\pi M^5}\right]\cos\theta. \tag{31}$$

As shown in Eqs. (28)–(31), both the metric and scalar field are regular at $r \to \infty$ and at $r = 2M$. From Eqs. (27), (28), and (30), we also find that the higher-order correction effectively reduces the frame-dragging effect around the black hole compared with the first-order approximation of the Kerr black hole. This result provides contrast to the result obtained in Refs. 8) and 9) within the Jackiw-Pi model. The reduction of the frame-dragging effect is attributed to the existence of the kinematic term of the scalar field.

Observational implications We discuss observational implications for the black hole solution given in Eq. (27). The CS correction in $g_{t\phi}$ has the same polar angle dependence as that of the first-order rotational approximation of the Kerr solution. Thus, the CS correction can be found only from the radial dependence of $g_{t\phi}$. The CS correction is composed of higher-order terms of $1/r$, i.e., at least terms of $O\left(1/r^4\right)$. This fact denotes that the CS correction effectively modifies the frame-dragging effect in the vicinity of the black hole. Therefore, to provide an upper limit for the CS coupling, it is useful to observe an object passing close by the black hole. Let us consider a massive particle with mass m and four-momentum p^μ on the equatorial plane of the black hole spacetime. For a particle with energy $E = -p_t/m$ and angular momentum $L = p_\phi/m$, from $p^\mu p_\mu = -m^2$, we obtain

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 - V^2(r), \tag{32}$$

where τ is the proper time of the particle, and

$$V^2 = \left(1-\frac{2M}{r}\right)\left(1+\frac{L^2}{r^2}\right)+LEJ\left[\frac{4}{r^3}-\tilde{\ell}^2\frac{1}{Mr^8}\left(70r^2+120Mr+189M^2\right)\right]. \tag{33}$$

Here, we define $\tilde{\ell}^2 \equiv \ell^2/896\pi$. Particle trajectories are permitted for a region where $E^2 > V^2$. Figure 1(a) shows the potential under a certain condition with the CS

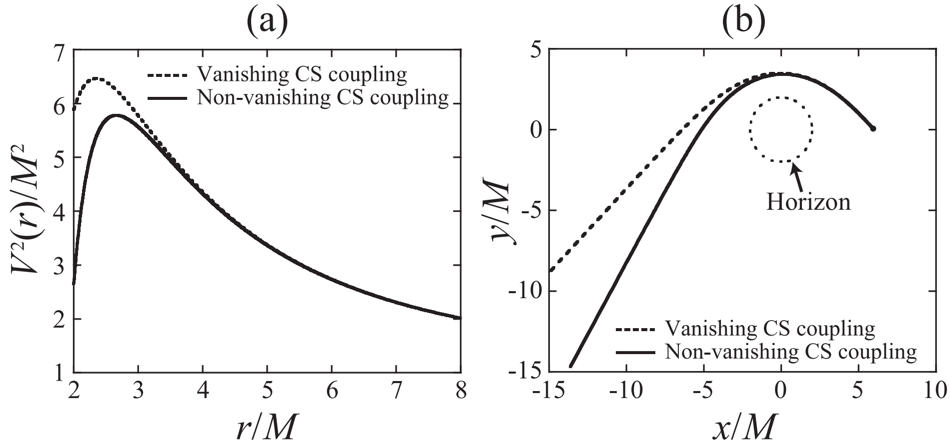

Fig. 1. (a) Potential $V^2(r)$. (b) Particle trajectories in the equatorial plane. The starting point is indicated by a small filled circle. The results in the presence of the CS coupling are denoted by solid curves, while the results in the absence of the CS coupling are denoted by dashed curves. In both figures, we assume $E = 2.35$, $L/M = 10.0$, $J/M^2 = 0.5$, and $\tilde{\ell}^2 = 0.1$.

coupling (solid curve) and that without the CS coupling (dashed curve). In this figure, we assume corotating trajectories. The CS correction reduces the potential for a region close to the horizon $r = 2M$. At the peaks of the potentials, unstable circular orbits are realized if $E^2 = V^2(r_{\rm peak})$. As seen in Fig. 1(a), the peak is shifted outward by the CS correction. However, when a particle that has energy of $E^2 < V^2(r_{\rm peak})$ falls freely from the outside ($r > r_{\rm peak}$), the periastron is shifted inward by the CS reduction of the potential. Multiplying Eq. (32) by $(d\phi/d\tau)^{-2}$ and differentiating that with respect to ϕ, we obtain the differential equation for trajectories

$$\frac{d^2r}{d\phi^2} = \frac{1}{L^2}\left(2E^2r^3 - 2r^3 + 3Mr^2 - L^2r + ML^2\right) + \frac{2J}{L^3}\left[2Er^2 - E^3\frac{(3r-8M)r^3}{(r-2M)^2}\right]$$
$$+\tilde{\ell}^2\frac{J}{L^3}\left[E\frac{3(20r+63M)}{r^3} - E^3\frac{130r^2+189Mr-189M^2}{r^2(r-2M)^2}\right]. \tag{34}$$

We can obtain particle trajectories by solving this equation. Figure 1(b) shows trajectories for the potentials given in Fig. 1(a). We used the Runge-Kutta method to integrate Eq. (34) numerically. As expected, we find that the periastron is closer to the black hole in the presence of the CS coupling than its absence. As seen from Fig. 1, the CS correction effectively modifies the trajectory around the periastron. The effect would also be magnified at the apastron. Therefore, observations of a satellite in an extended elliptical orbit, for example, a system such as OJ287,[17] would provide useful information about the CS coupling.

Concluding remarks We have investigated a rotating black hole in the extended Chern-Simons (CS) modified gravity. We considered the perturbation of the Schwarzschild spacetime with a vanishing scalar field to discuss the rotation of a black hole. The assumption of the vanishing scalar field at the zeroth order allows us to compare a result with that of the Kerr solution. By taking account of the scalar field that is purely excited by a gravitational field through the Chern-Pontryagin density, we obtained an approximate solution for a slowly rotating black hole. In obtaining the solution, we adopt both the slow rotation approximation and the approximation of a weak CS coupling. Consequently, we found that the CS correction gives an effective reduction of the frame-dragging around a black hole in comparison with that of the Kerr solution. We also showed that observations of systems that have a satellite in an extended elliptical orbit would be useful to provide an upper limit for the CS coupling. As a future work, it would be interesting to seek exact solutions for a rotating black hole in the extended CS modified gravity. From such investigations, the nonlinear characteristics of the CS modified gravity would also be revealed.

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1) J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998).
2) C. Rovelli, Living Rev. Relativity **11** (2008), 5.
3) C. L. Bennett et al., Astrophys. J. Suppl. **148** (2003), 1.
4) R. Jackiw and S.-Y. Pi, Phys. Rev. D **68** (2003), 104012.
5) T. L. Smith, A. L. Erickcek, R. R. Caldwell and M. Kamionkowski, Phys. Rev. D **77** (2008), 024015.
6) S. Deser, R. Jackiw and S. Templeton, Ann. of Phys. **140** (1982), 372.
7) M. Adak and T. Dereli, arXiv:0807.1832.
8) K. Konno, T. Matsuyama and S. Tanda, Phys. Rev. D **76** (2007), 024009.
9) K. Konno, T. Matsuyama, Y. Asano and S. Tanda, Phys. Rev. D **78** (2008), 024037.
10) D. Grumiller and N. Yunes, Phys. Rev. D **77** (2008), 044015.
11) D. Grumiller, R. Mann and R. McNees, arXiv:0803.1485.
12) N. Yunes and C. F. Sopuerta, Phys. Rev. D **77** (2008), 064007.
13) S. Alexander and N. Yunes, Phys. Rev. D **75** (2007), 124022; Phys. Rev. Lett. **99** (2007), 241101.
D. Guarrera and A. J. Hariton, Phys. Rev. D **76** (2007), 044011.
B. Tekin, Phys. Rev. D **77** (2008), 024005.
N. Yunes and D. N. Spergel, arXiv:0810.5541.
K. K. Nandi, I. R. Kizirgulov, O. V. Mikolaychuk, N. P. Mikolaychuk and A. A. Potapov, arXiv:0812.4342.
14) S. Weinberg, Phys. Rev. D **77** (2008), 123541.
15) T. Regge and J. A. Wheeler, Phys. Rev. **108** (1957), 1063.
16) See, for example, Y. L. Luke, *The Special Functions and Their Approximations, 2 Vols.* (New York, Academic Press, 1969).
17) M. J. Valtonen et al., Nature **452** (2008), 851.
18) N. Yunes and F. Pretorius, arXiv:0902.4669.

Note added: The same solution for the metric was also derived independently in Ref. 18).