Topological Crystals: as a New Paradigm

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Outline

(1) Introduction

(2) Crystal Growth   Covalent wires

(3) Topological Crystals

Ring, Möbius, Figure8, Hopf-link Crystals

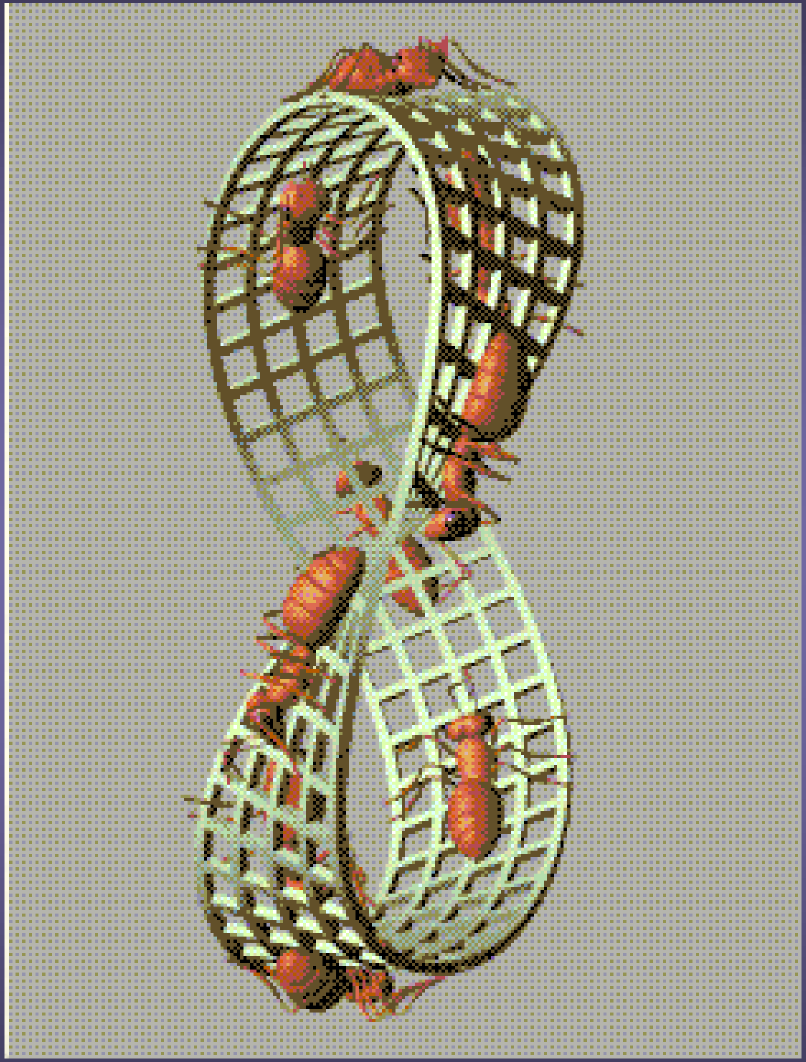
Mechanism

(4) Classification of Topological Crystals

(5) Electronic properties: AB effect, Frolich supercon.

(6) Summary

Introduction: Escher's ants



Möbius Strips II (1963)
by M. C. Escher

Ants return to the original point after two laps on Möbius Strips

No distinction between the Obverse and the Reverse

↓

An exotic one-sided world !!

↑

from Artists to Scientists

Ants ⟶ Electrons, Spins
Photons, CDWs, Super

Strips ⟶ Lattice, Crystal

Is it Possible ?

What happens ?

Our Goal

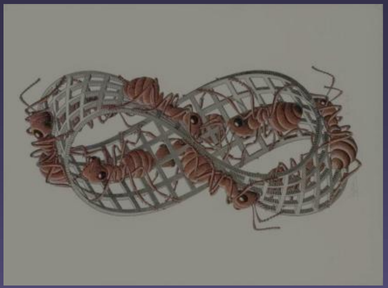
1. Topological Crystallography (Ladder)

2. Quantum topological effects in the topological crystals (Ants)

Macroscopic quantum coherence in nontrivial topological space

Topological gauge fields

Strategy **Is it Possible ?**



For synthesize of topological crystals

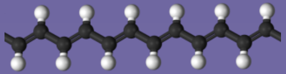
Requirements

★ One-dimensional wire

Organic polymer

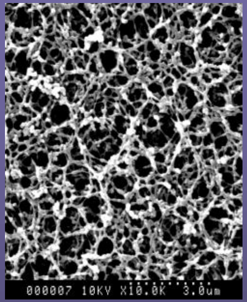
Protein

DNA

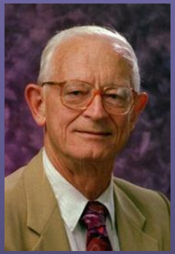


$(CH)_x$

Too flexible



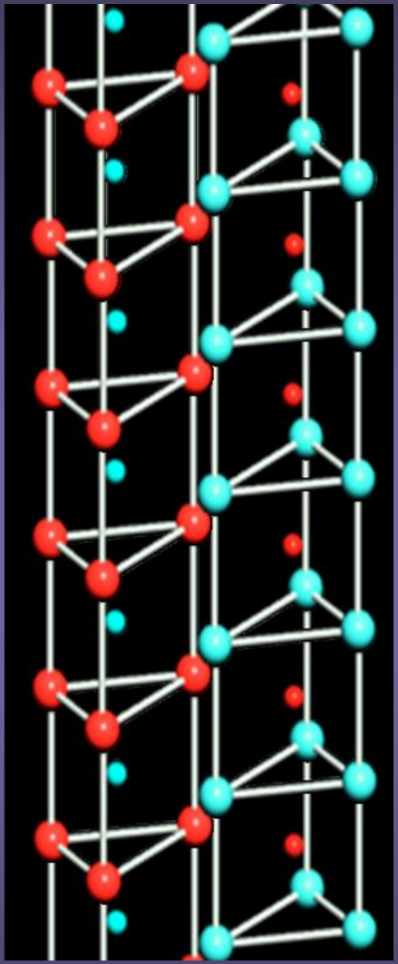
$(SN)_x$



★ Rigidity

Inorganic polymer ⟶ MX_3, MX_2

Inorganic Covalent wires



Requirement for twisting and bending materials such as Mobius crystals

chain ↑

Semi-flexible !

Typical materials

NbS_3 $NbSe_3$

TaS_3 $TaSe_3$

TiS_3 $ZrTe_3$

⋮ ⋮

MX_3

Covalent Intrachain

Van der waals force: Inter-chain

MX_3 : CDW and Superconductors

Researches in Hokkaido Univ. 35 Years

Discovery of $NbSe_3$ and TaS_3: Yamaya, Sambongi, Tsutsumi (1977)

Memory Effect of CDW in $NbSe_3$: Ido Oda, Okajima, Sambong (1986)

Ring Crystals in $NbSe_3$: Kawamoto, Okajima, Yamaya, Tanda (1999)

Möbius Crystals in $NbSe_3$ Tanda Tsuneta Inagaki Okajima Yamaya Hatakenaka

Nature 417 397 (2002)

Hopf-link crystals Matsuura, Yamanaka, Hatakenaka, Matsuyama, Tanda

PRB (2006)

Topological quantum Effects in MX_3: Tsuboa, Matsuura, Kumagai, Tanda

PRB (09,10,11,12)

Chiral CDW in $TiSe_2$: Ishioka, Oda, Ichimura, Tanda **PRL (2010,2012)**

MX_2 Nanotubes (03,04), Topology change(09,10,11,12)

Ring Crystals

Seamless!!

1999

Synthesize and Condition

Chemical Vapor Transportation

Closed Quartz Tube

<10^{-7}Torr(initial vacuum)

10 days

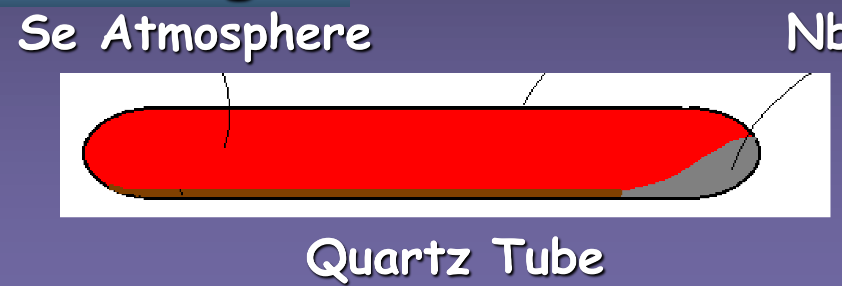
Nb,Ta (99.999%),

Se (99.9999%)

Furnace Temperature 600℃~800℃

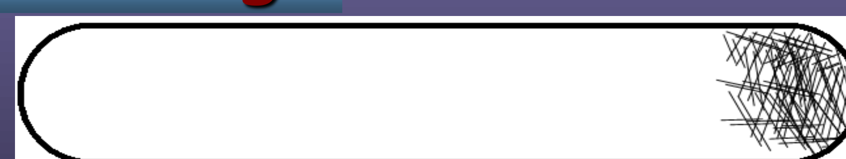
Starting materials (Nb, Se) is sealed in an evacuated ampoule.

1. heating



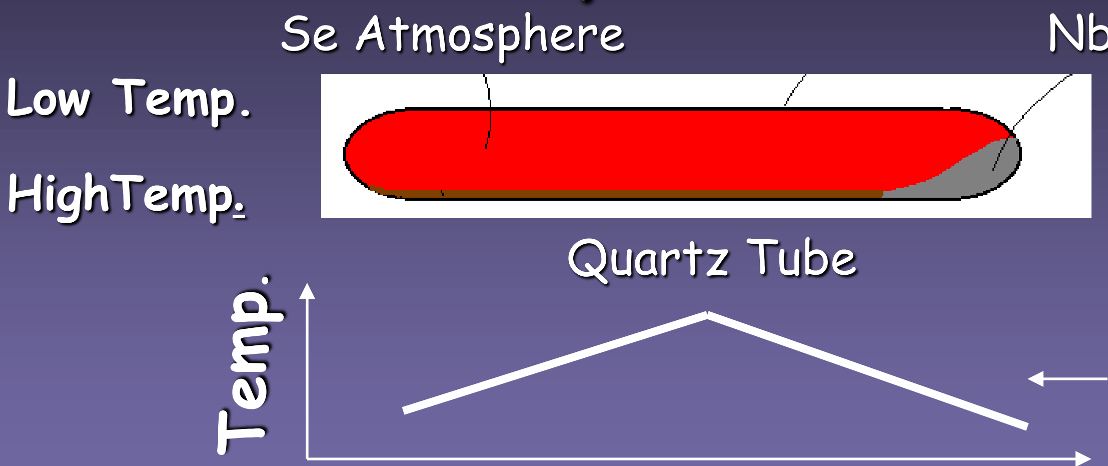
2.transportation

3.final stage



After a few weeks of heating, products are transported to the lowest-temperature end where vapor pressure is also the lowest.

Nonequilibrium condition

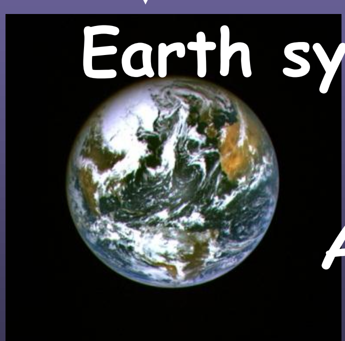


Large Temperature gradient (more than 150℃)

Se circulates through

Vapor , mist , liquid droplet

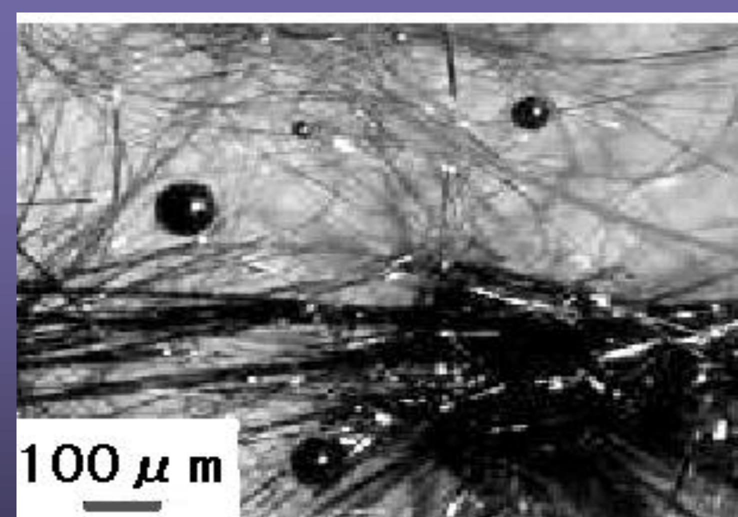
↓



Earth system make

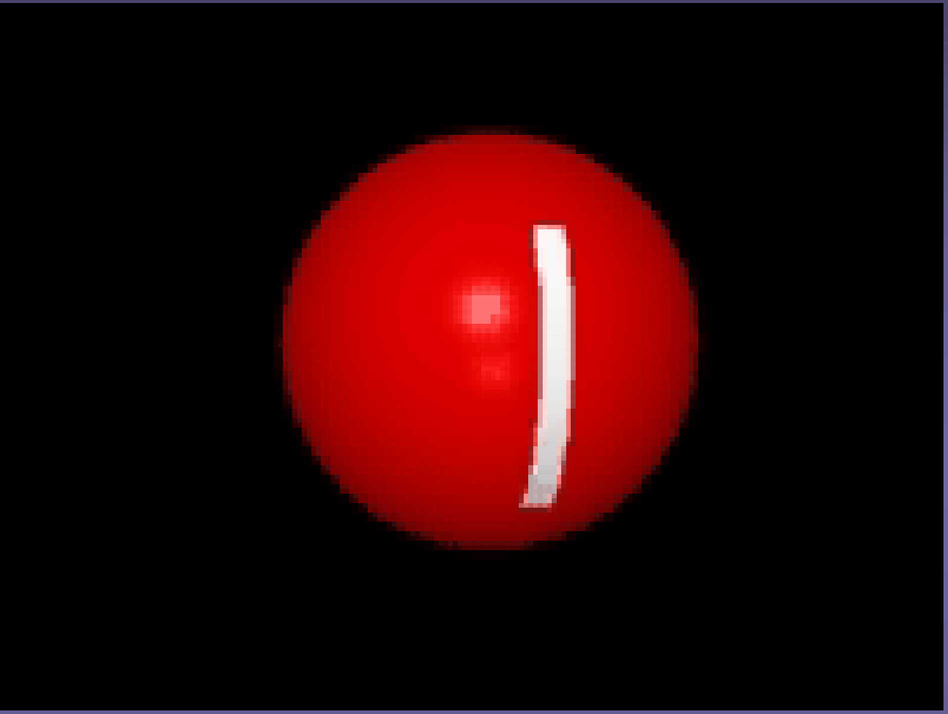
Life polymer

Analogous method

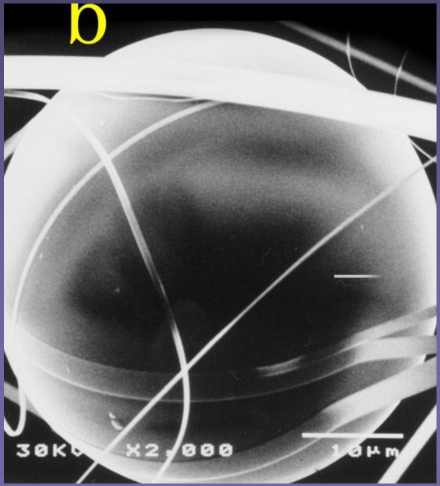


These droplet need for formation of ring crystals.

Formation of Ring Crystals



a growing crystal can eat its own tail.

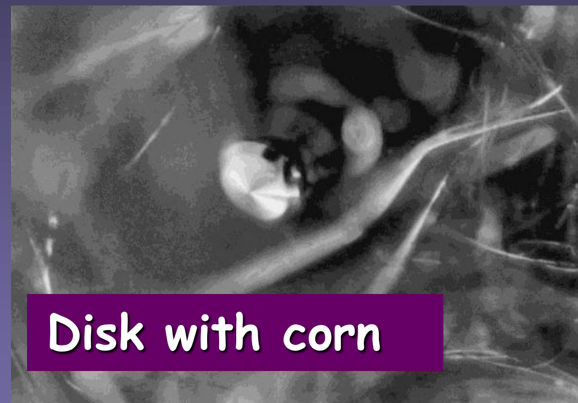


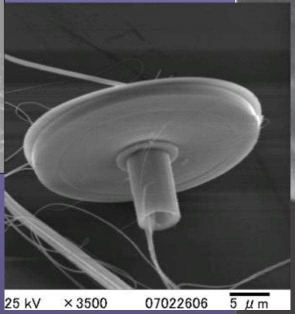
The ribbon-shaped $NbSe_3$ crystals grown in the viscous Se droplet are bent due to Se surface tension.

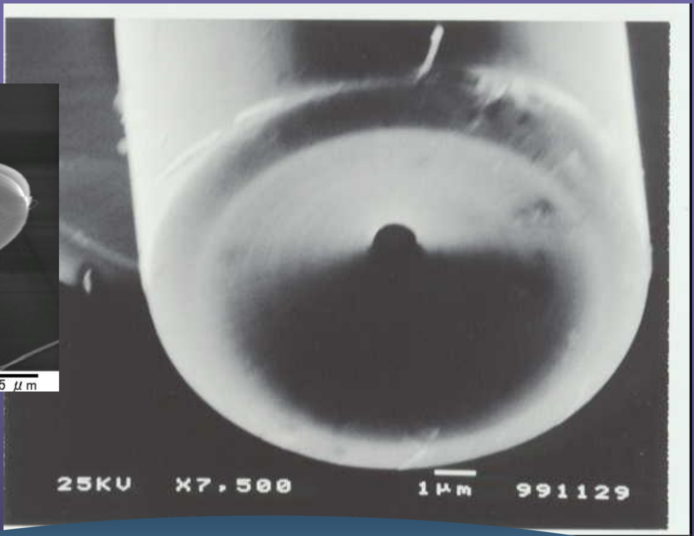
NbSe3 fiber circulate on the equator of Se droplets during growth

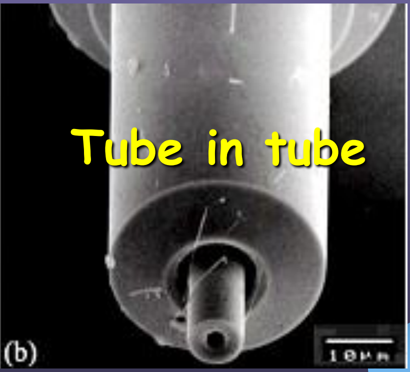
Seamless ring

Disks and Tubes Crystals









Hole radius is less than 1 μm !!

CDW Correlation length is 5 μm , which is larger than hole radius!

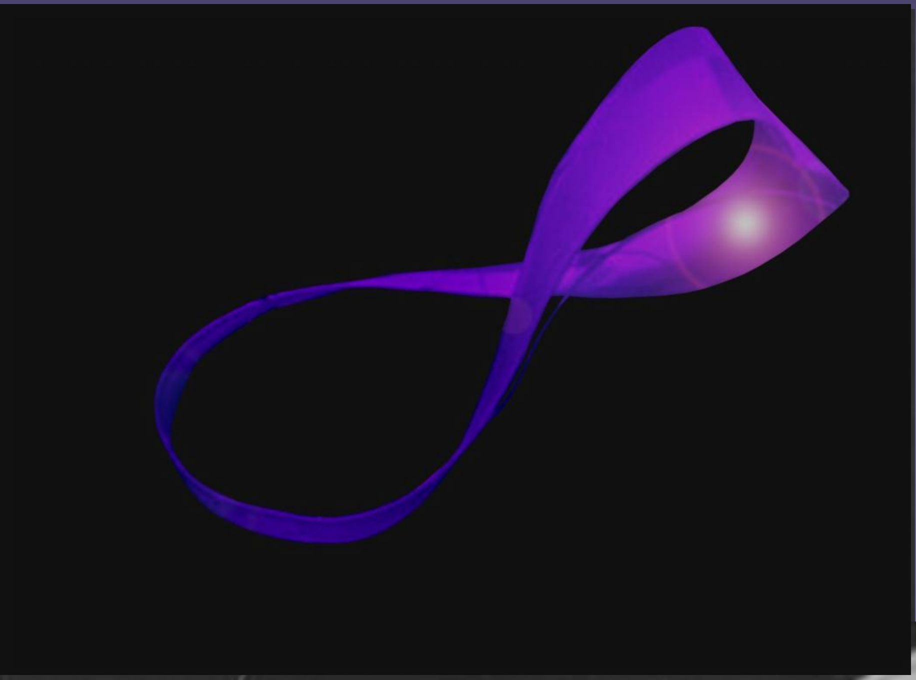
This system is ideal matter in observation of Interference effect or AB effect of CDW

Figure-8 Crystals (2π-twist)

Knot with Framing
in Math. term

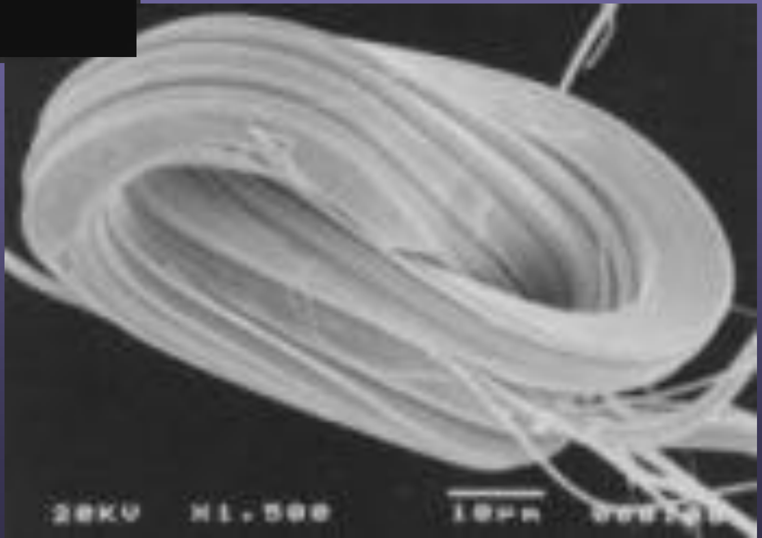
+2π-Twist -Band
−2π-Twist -Band

No evidence of breaking
of Chiral Symmetry



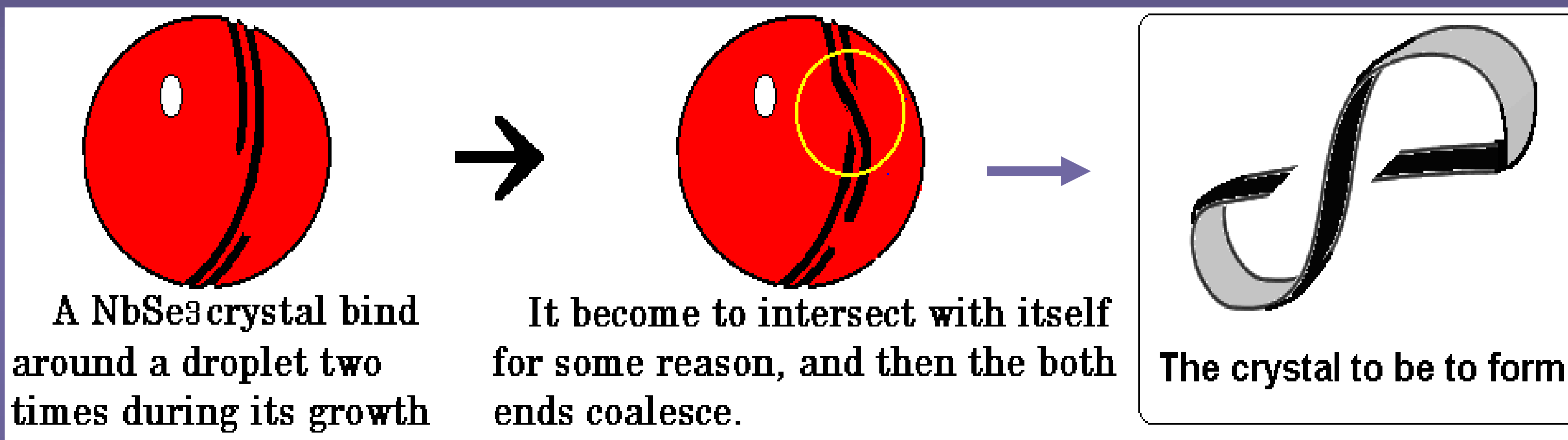
Growing

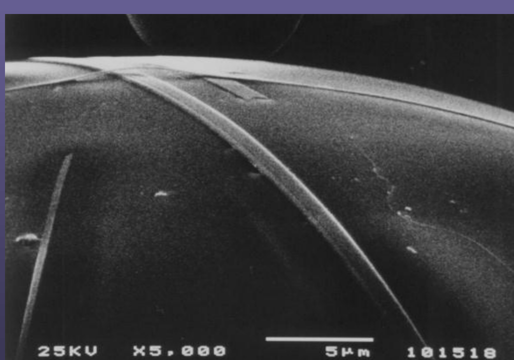
Self-crossing
Maximum Crystals

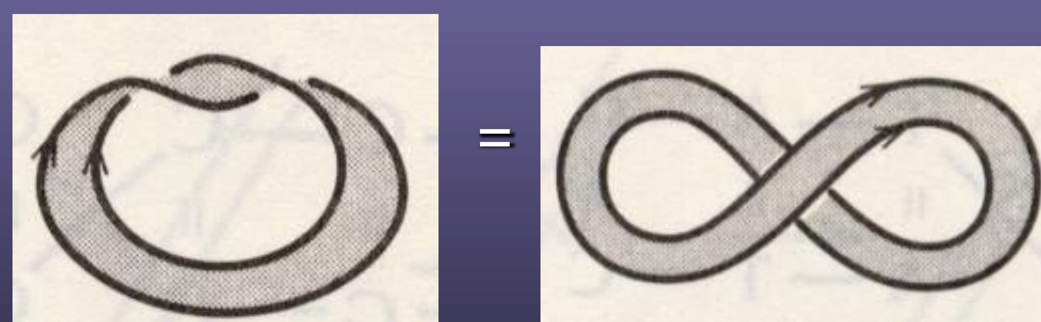


2002 Nature

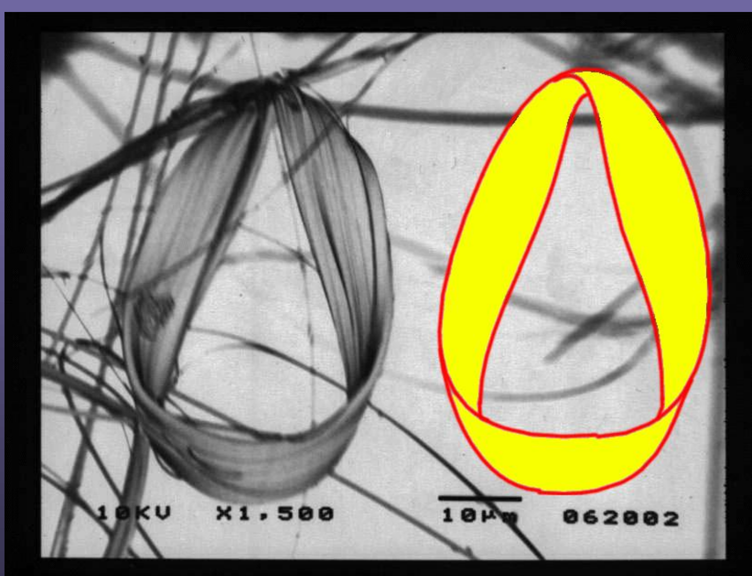
Mechanism of formation of Figure 8 Crystals (2π twist)



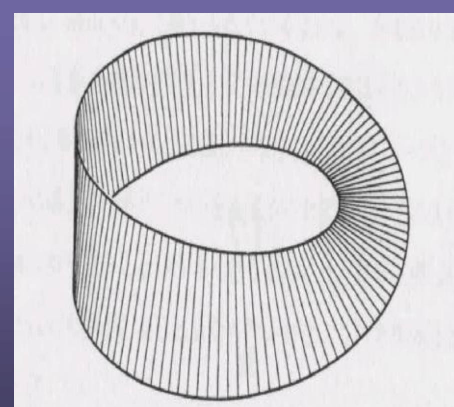


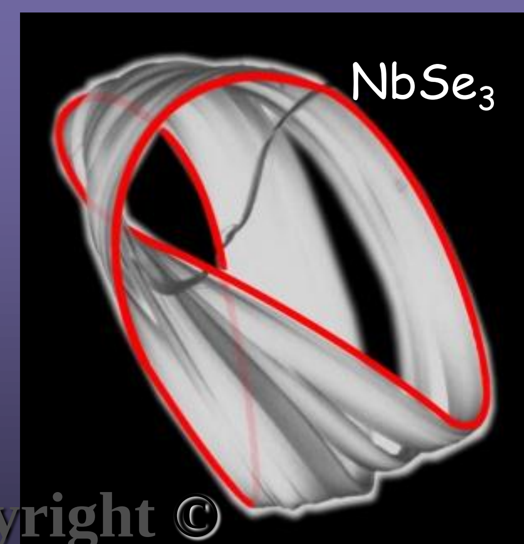


Möbius Crystals (π twist)



TaSe3





Bending to Twisting

formation mechanism of Mobius crystals

Difficulties → twisting shear

Question is how the twisting is introduced without actual shear forces

Answer

Crystal symmetry ⇒ The bending-twisting conversion

Bending ⇔ Twisting

Compliance

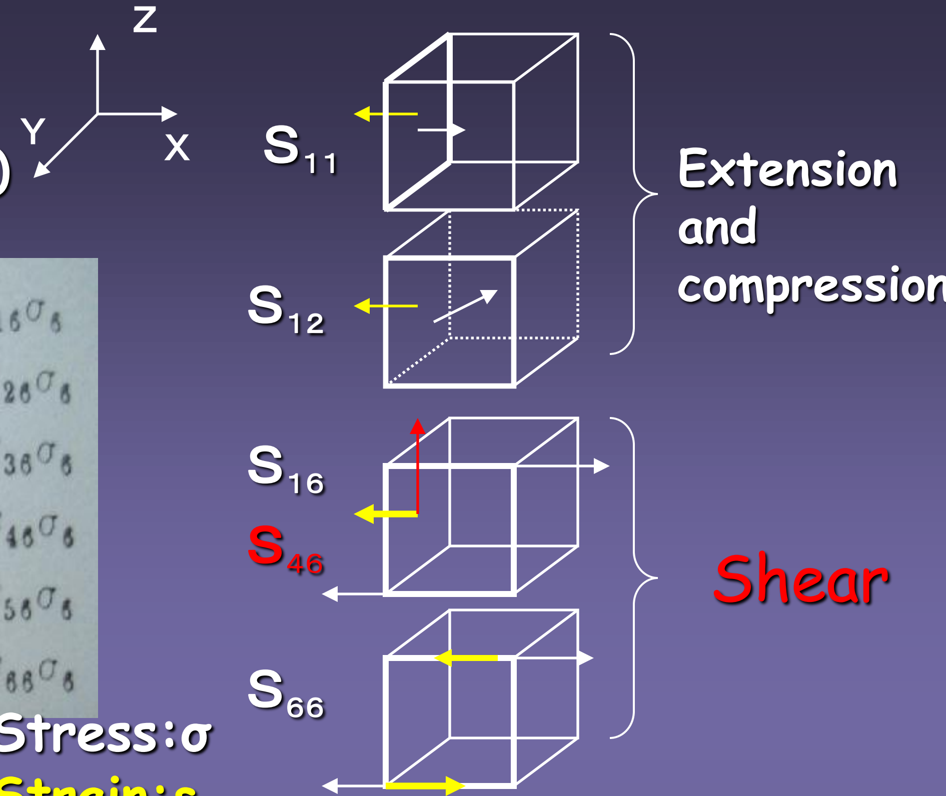
Monoclinic , triclinic

Compliance

Elasticity (Expansion of Hook's law)

$X=(1/K)\times F$ $F=kX$

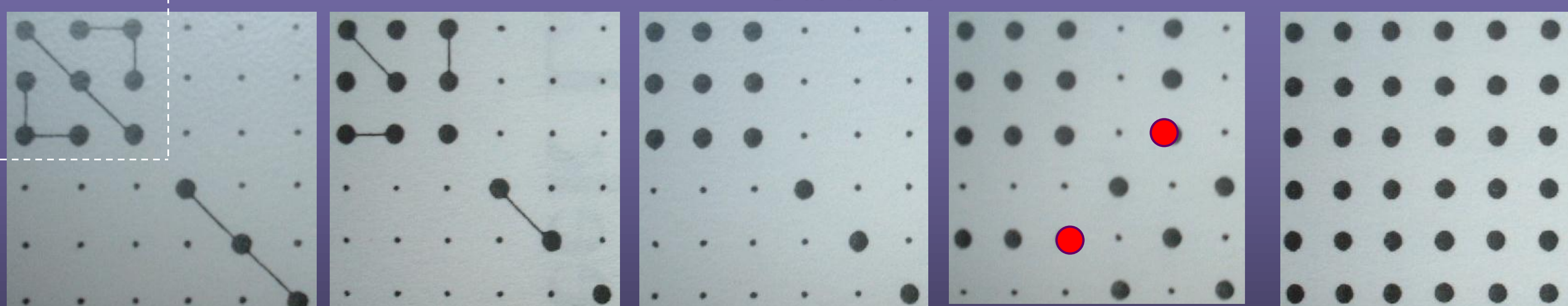
$$
\begin{aligned}
\epsilon_1 &= s_{11}^{E,T}\sigma_1 + s_{12}\sigma_2 + s_{13}\sigma_3 + s_{14}\sigma_4 + s_{15}\sigma_5 + s_{16}\sigma_6 \\
\epsilon_2 &= s_{12}\sigma_1 + s_{22}\sigma_2 + s_{23}\sigma_3 + s_{24}\sigma_4 + s_{25}\sigma_5 + s_{26}\sigma_6 \\
\epsilon_3 &= s_{13}\sigma_1 + s_{23}\sigma_2 + s_{33}\sigma_3 + s_{34}\sigma_4 + s_{35}\sigma_5 + s_{36}\sigma_6 \\
\epsilon_4 &= s_{14}\sigma_1 + s_{24}\sigma_2 + s_{34}\sigma_3 + s_{44}\sigma_4 + s_{45}\sigma_5 + s_{46}\sigma_6 \\
\epsilon_5 &= s_{15}\sigma_1 + s_{25}\sigma_2 + s_{35}\sigma_3 + s_{45}\sigma_4 + s_{55}\sigma_5 + s_{56}\sigma_6 \\
\epsilon_6 &= s_{16}\sigma_1 + s_{26}\sigma_2 + s_{36}\sigma_3 + s_{46}\sigma_4 + s_{56}\sigma_5 + s_{66}\sigma_6
\end{aligned}
$$



S_{ab}:Compliance (tensor)

Stress:σ

Strain:ε



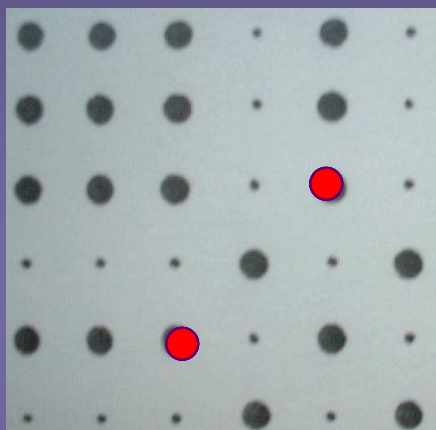
Cubic Tetragonal Orthornbic Monoclinic Triclinic

→ Lower symmetry

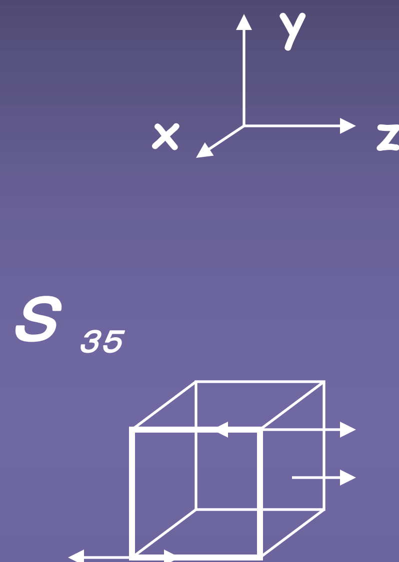
・: zero

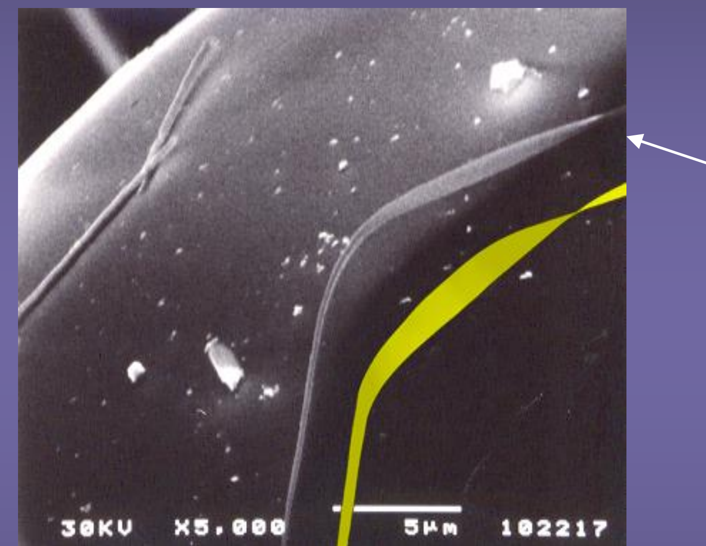
●: nonzero

Evidence of Twisting



Monoclinic NbSe3

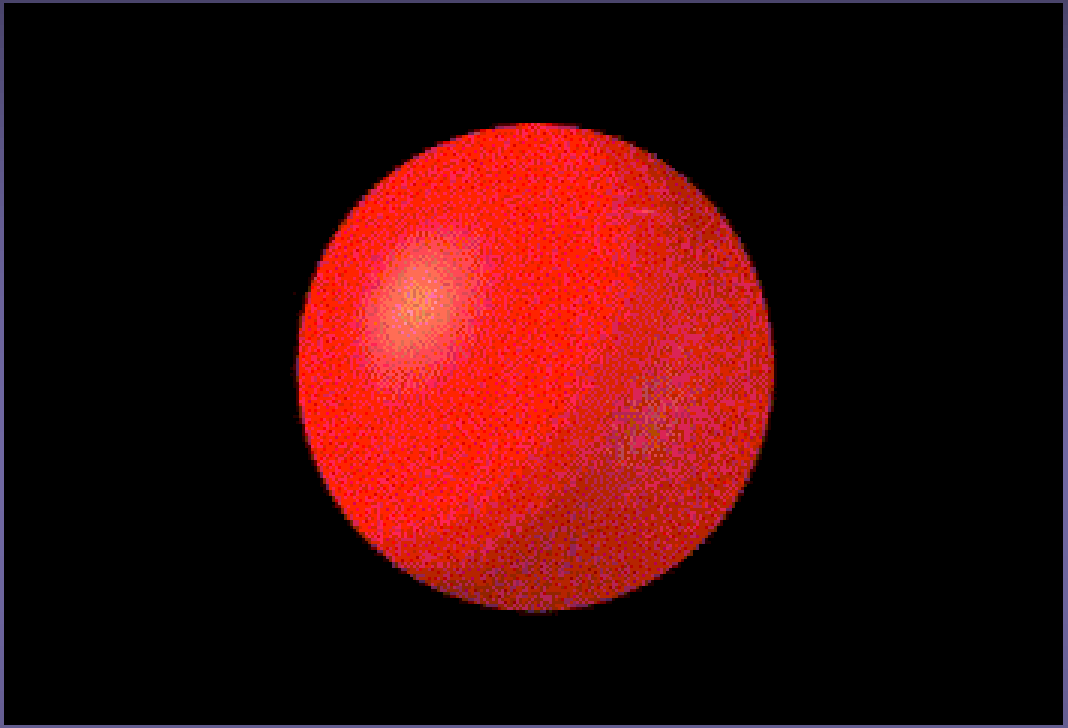


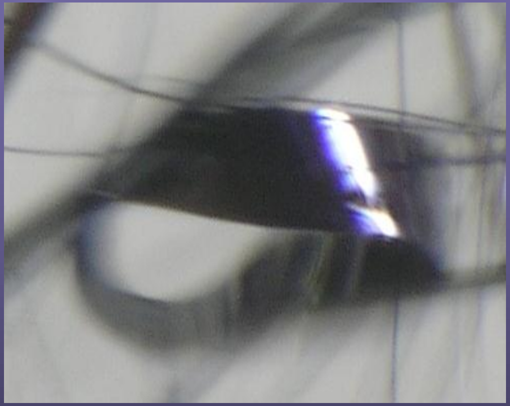


▪: zero
●: nonzero
●: Shear term for Monoclinic

twisting is introduced by bending through this mechanism without actual shear forces during the growth processes along surface

Formation of Möbius Crystals





To form a Möbius strip (1π) , a twist is needed to be introduced during spooling.

Can we introduce the concept of topology into crystals ?

Originally, What's Crystal ?

↓

Bragg reflection !

Definition of International Union of Crystallography (1991)

Definition of Crystals

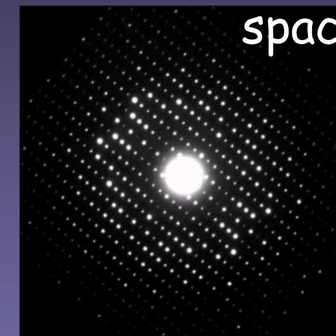
Crystals show Bragg reflection

Snow crystal

Bragg reflection of electron beam/X-ray

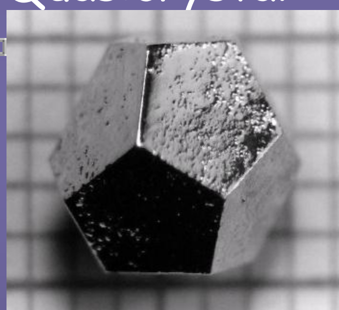
Mathematically, Fourier transform from real space (lattice) to *k*-space (Bragg spots)

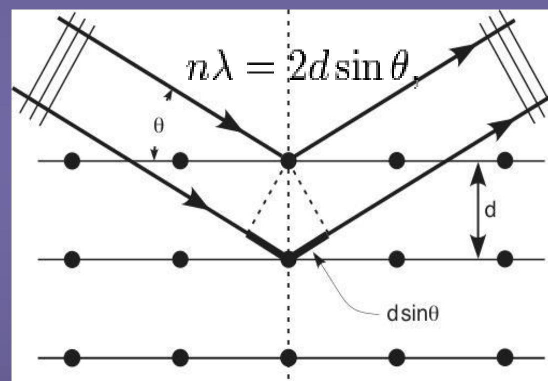
Reciprocal space

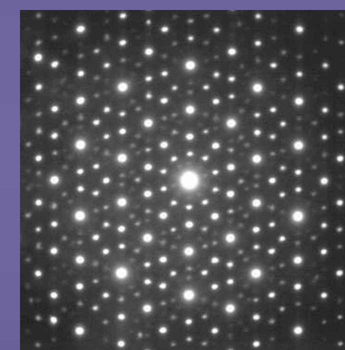


spot

Quasicrystal

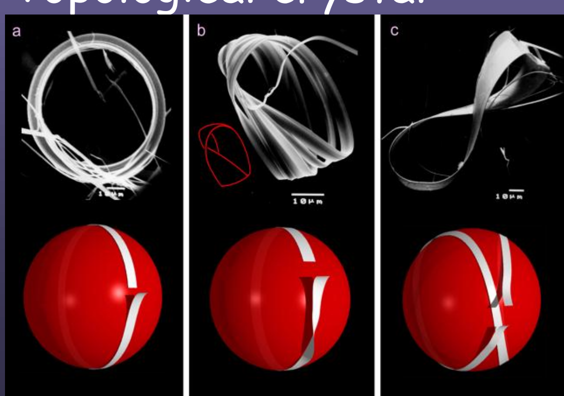






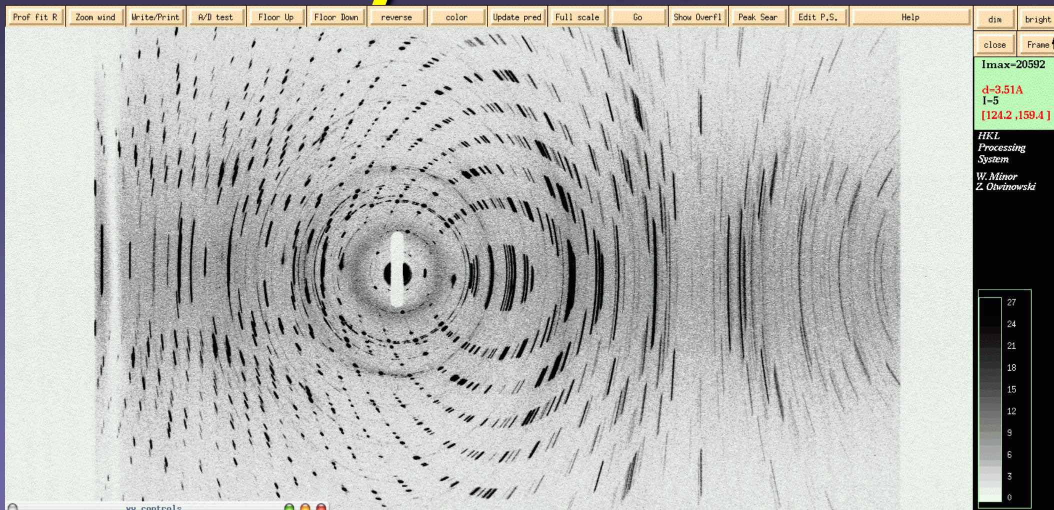
Crystals and quasicrysta
show sharp Bragg spots

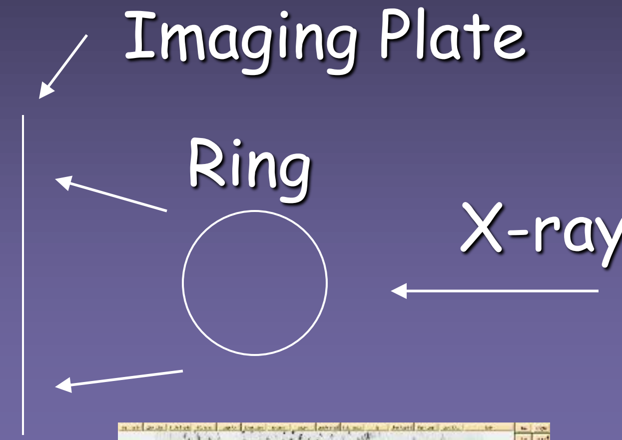
Topological crystal

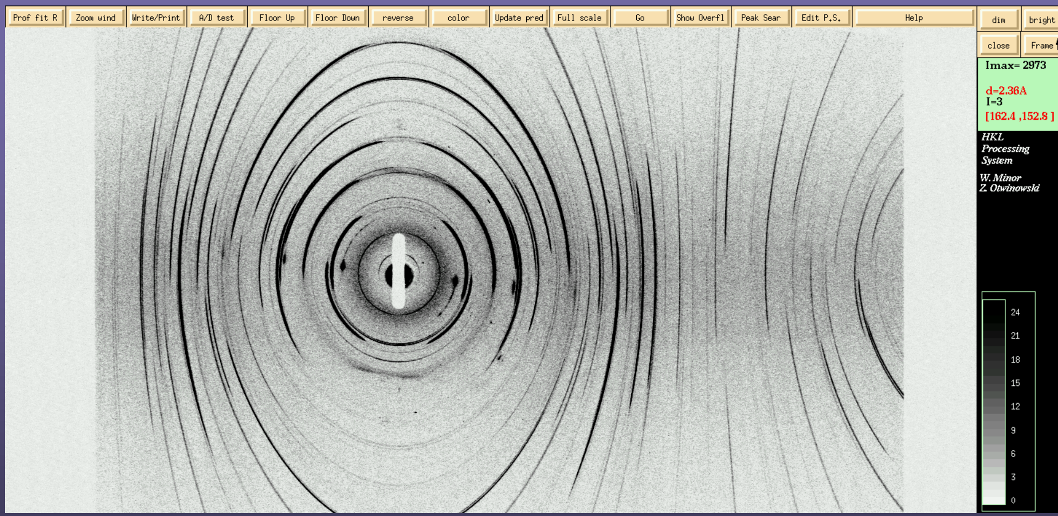


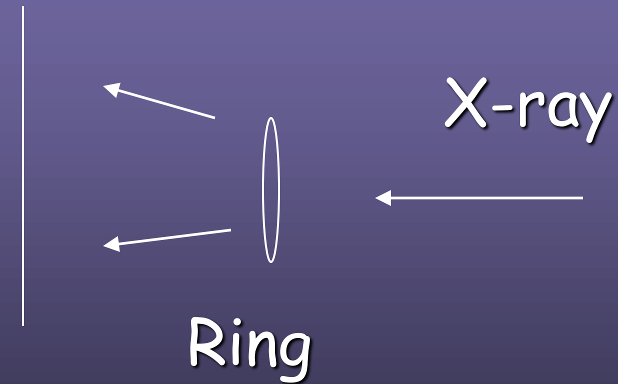
?

X-ray Diffraction: Ring







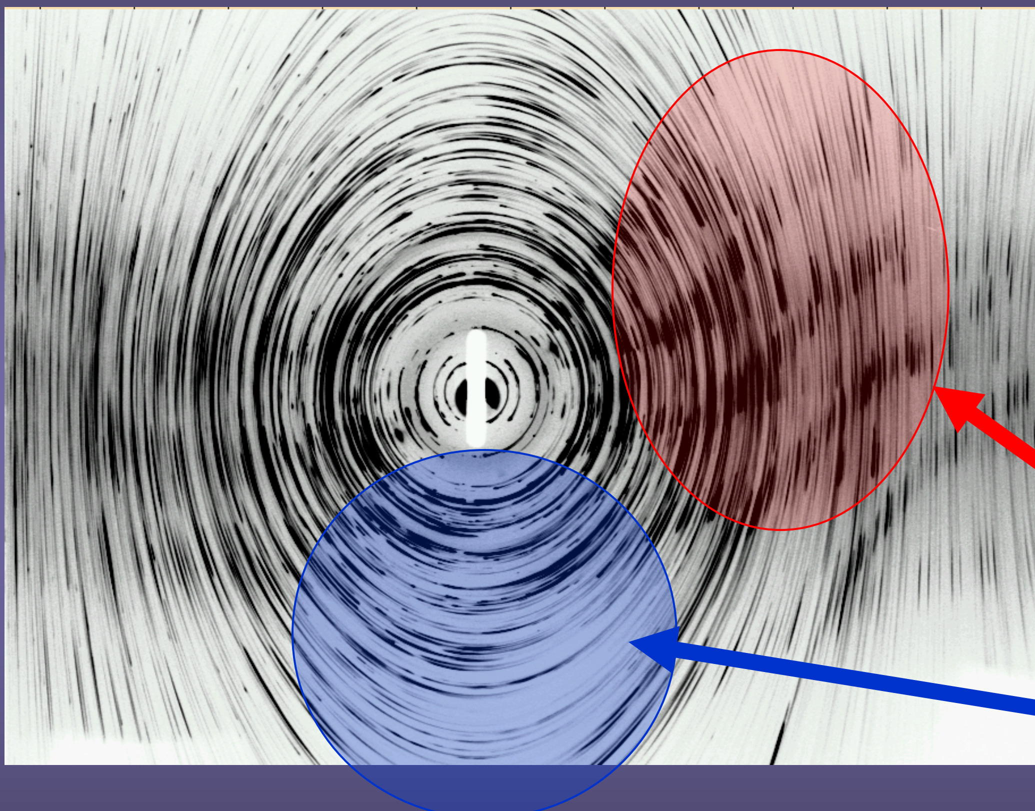


Bragg Ring

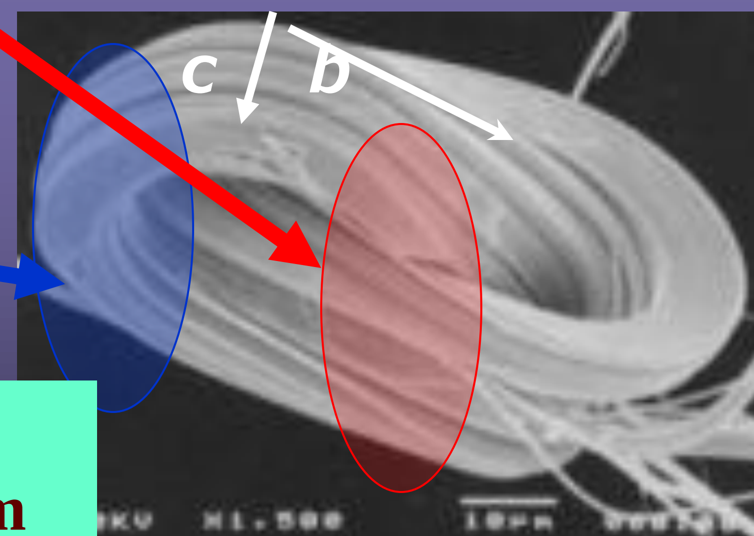
Line is homogeneous

Single Crystals !!

Texture pattern in 8 crystal



Short strong arcs are diffracted from semi-straight parts

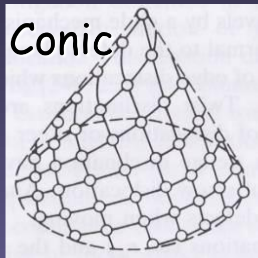


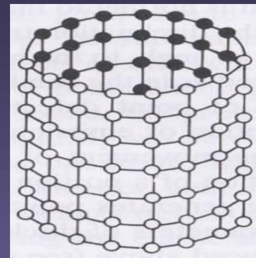
K-space

Long weak arcs are diffracted from hair-pin curves

Real space

Classification of Topological Crystals





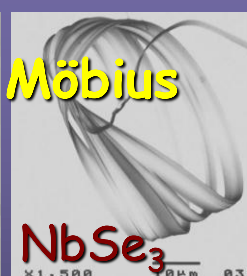
$\omega = 1/2\pi$
$\omega^* = 0\pi$

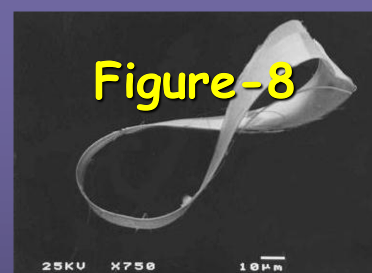
$\omega = 2\pi$
$\omega^* = 0\pi$

ω: Wedge Disclinations
ω^* : Twist

ω^*, ω is independent, respectively, due to the topological defect theory of the crystals, as yet. (by Frank)

Topological Crystals ⟷ **New Definition !** Knot Theory ⟷ **$L_k = (\omega + \omega^*)/ 2\pi - 1$ Linking Number**





$\omega = 2\pi$
$\omega^* = 0\pi$
$L_k = 0$

$\omega = 2\pi$
$\omega^* = 1\pi$
$L_k = 1/2$

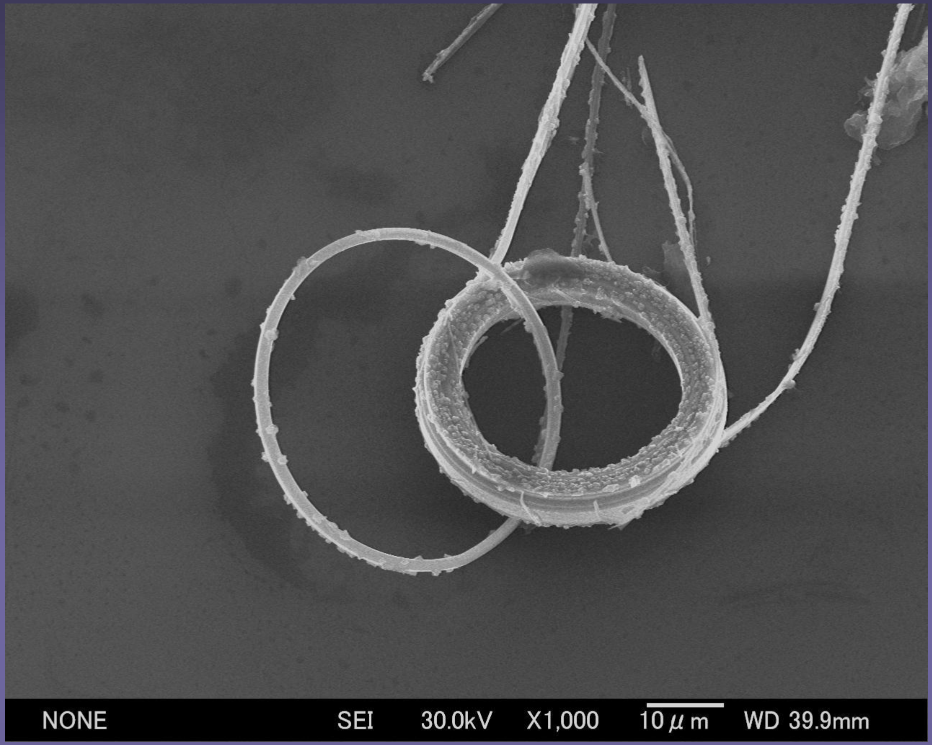
$\omega = 2\pi$
$\omega^* = 2\pi$
$L_k = 1$

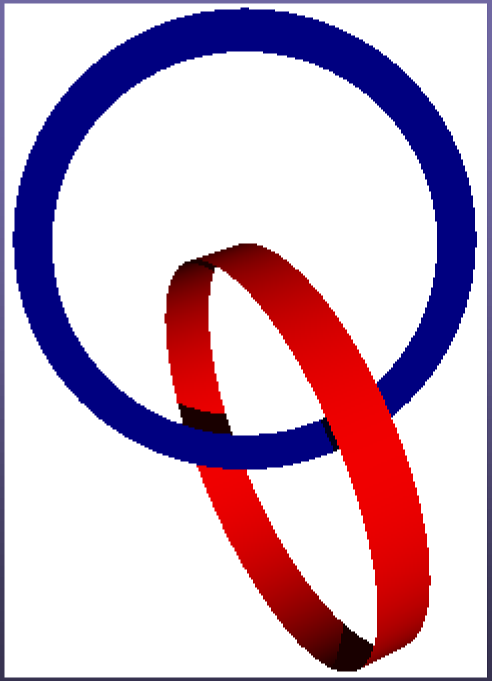
Global Wedge + Global Twisting

Knots Crystals !!

$$\frac{1}{2\pi}\int_{crystal} f\left(\omega,\omega^*\right) dV = L_k$$

Discover Hopf-Link Crystals



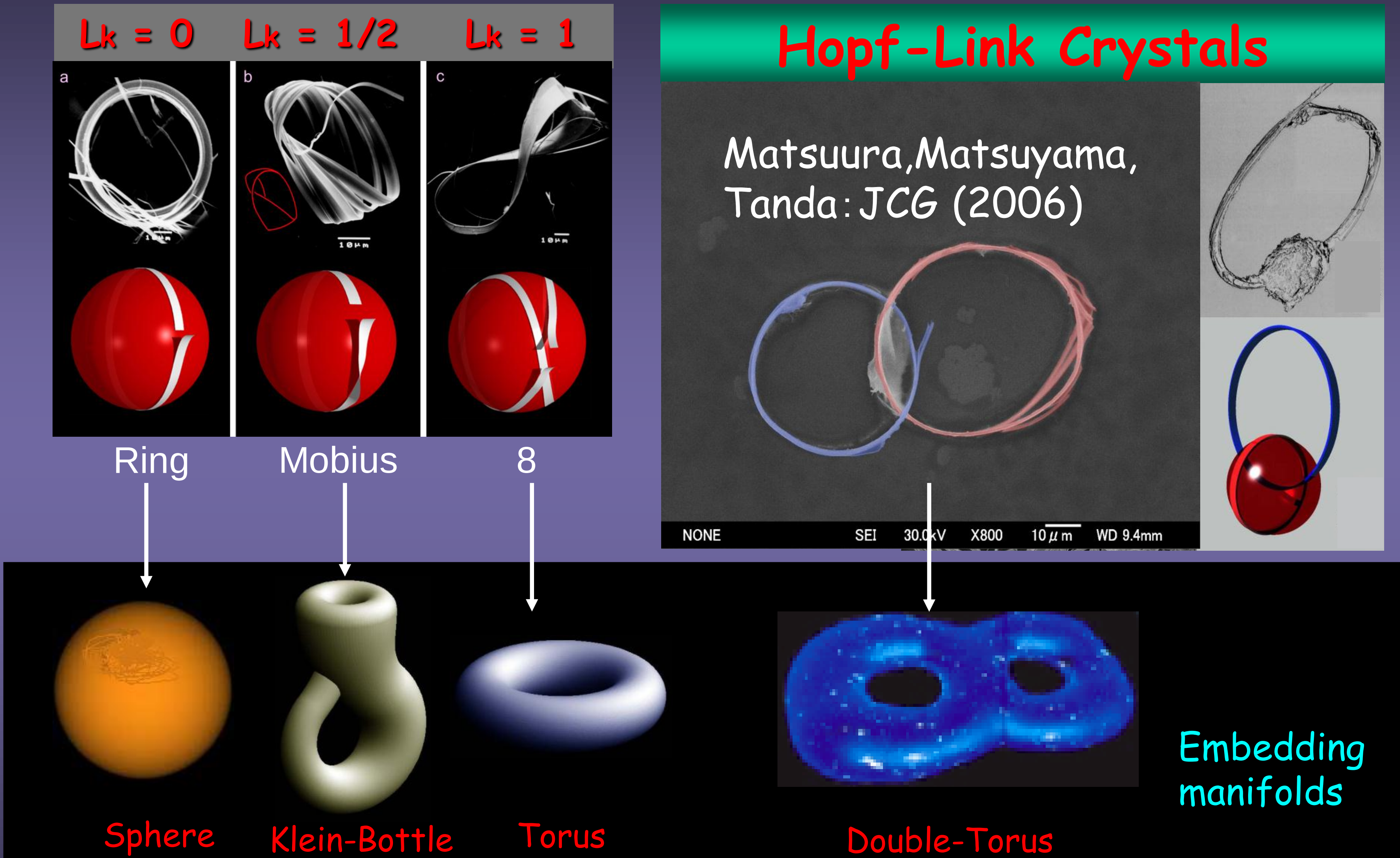


Topologically linked crystals

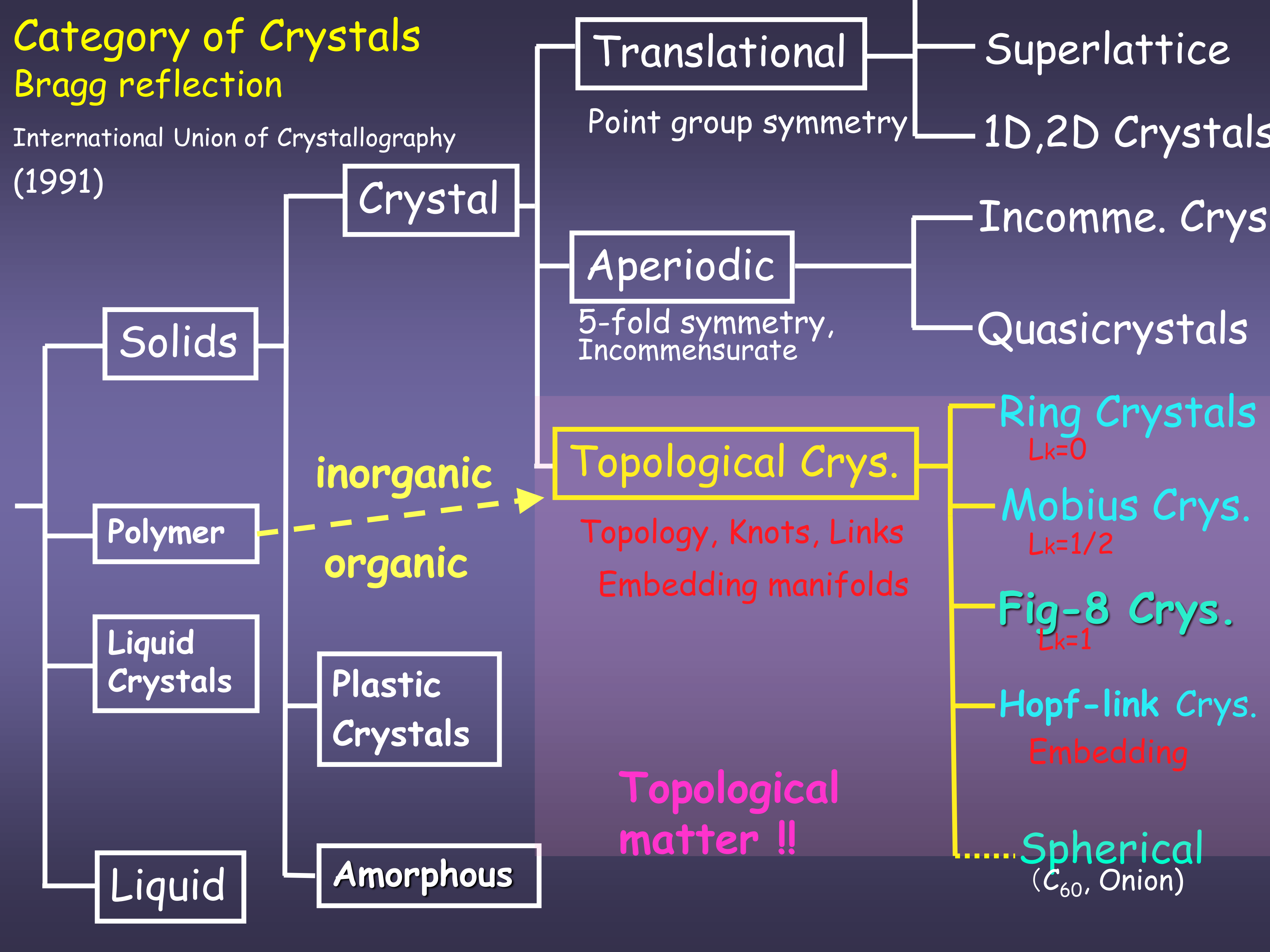
T.Matsuura, M.Yamanaka, N. Hatakenaka,T. Matsuyama, and S. Tanda, Journal of Crystal Growth 297, 157 (2006).

Hopf-link crystals cannot be categorized by the linking number of Knots crystals and usual point groups → New Classification

Classification By Embedding manifolds



We propose New Classification with Embedding manifolds



Category of Crystals
Bragg reflection
International Union of Crystallography
(1991)
Solids
Polymer
Liquid Crystals
Liquid
Crystal
Plastic Crystals
Amorphous
inorganic
organic
Translational
Point group symmetry
Superlattice
1D,2D Crystals
Aperiodic
5-fold symmetry, Incommensurate
Incomme. Crys
Quasicrystals
Topological Crys.
Topology, Knots, Links
Embedding manifolds
Ring Crystals
Lk=0
Mobius Crys.
Lk=1/2
Fig-8 Crys.
Lk=1
Hopf-link Crys.
Embedding
Spherical
(C60, Onion)
Topological matter !!

Our Goal

1. Topological Crystallography (Ladder)

↓

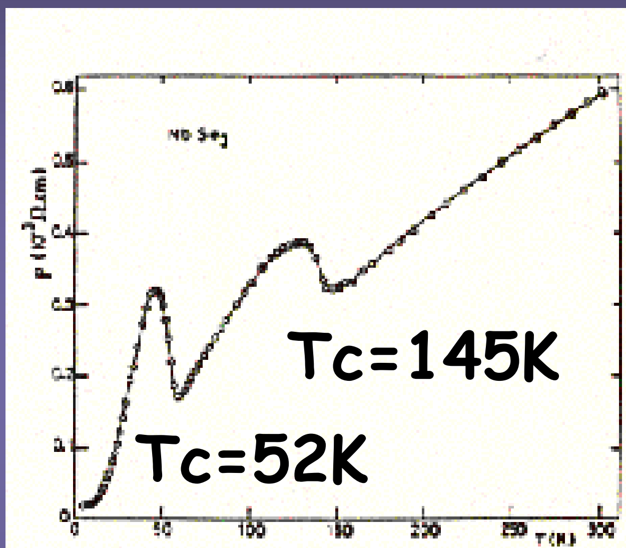
2. Quantum topological effects in the topological crystals (Ants)

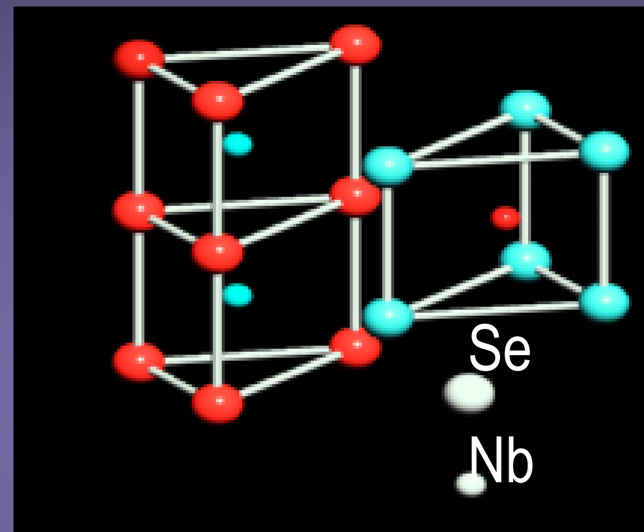
Macroscopic quantum coherence in nontrivial topological space

Topological gauge fields

$NbSe_3$: Charge Density Waves

CDW: Charge-Density-Waves

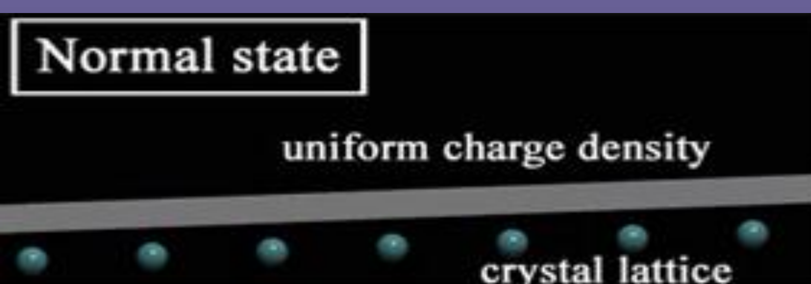




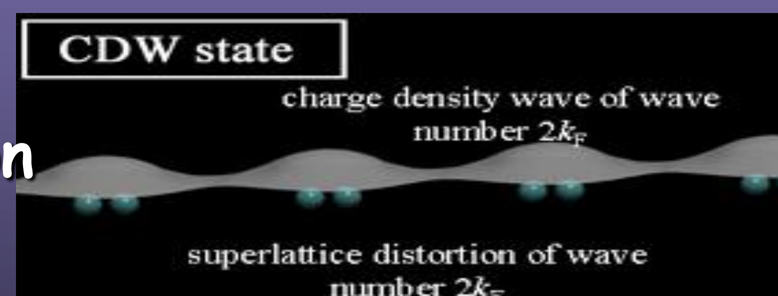
Chain axis

Covalent wire

Fiber structure



Transition

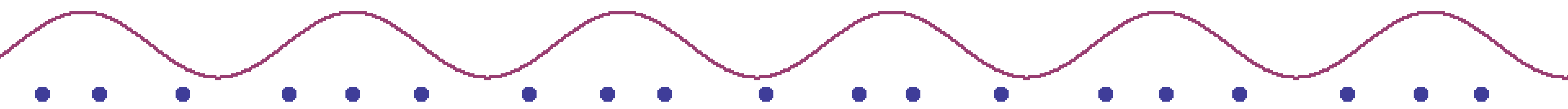


A periodic charge modulation

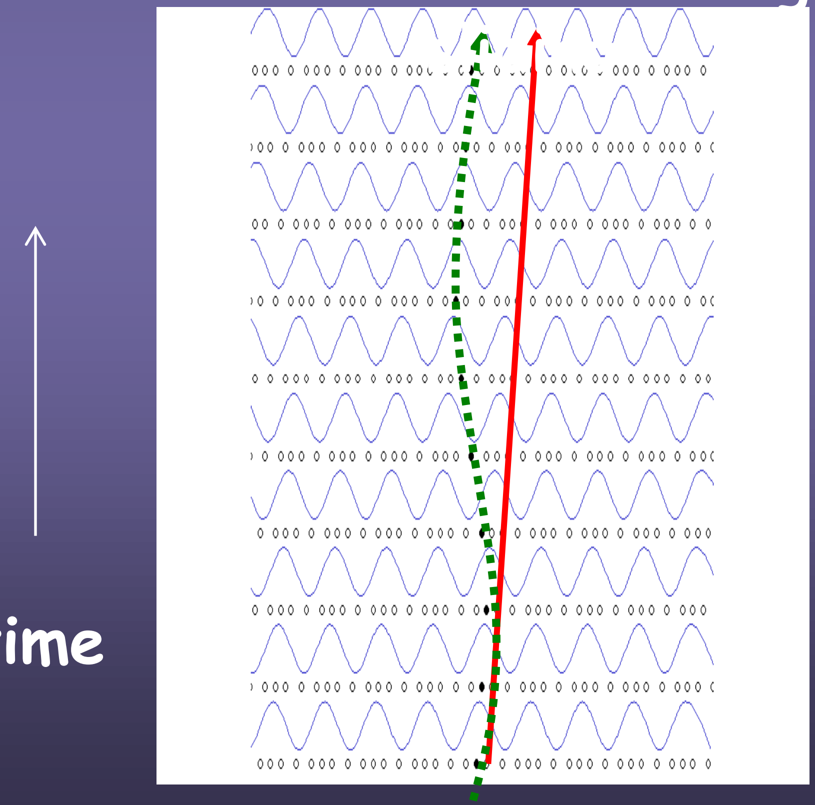
Macroscopic wave function

Sliding of Macroscopic wave function

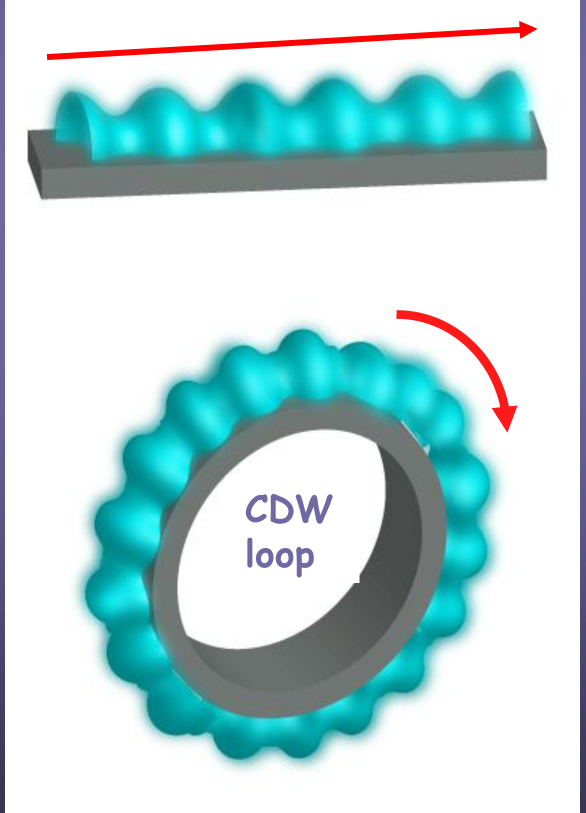
Fröhlich superconductors : phason



CDW sliding



time



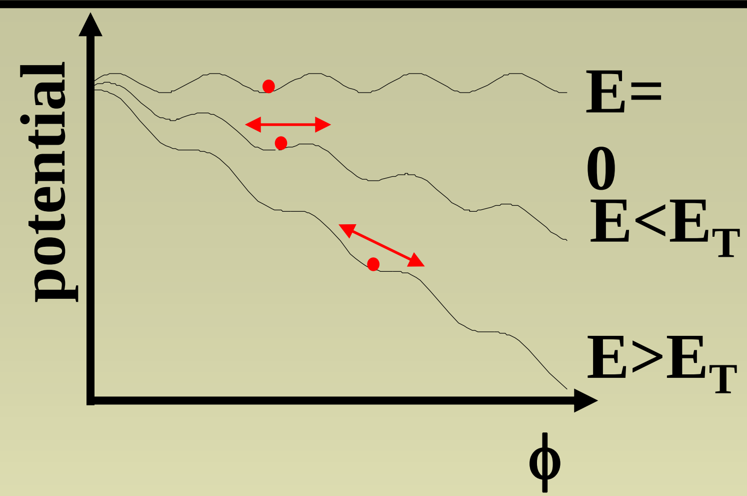
Eternal current

Real space topology

I. Circulating Sliding Current of CDW By Shapiro Steps in the Loop

II. AB-Effect of CDW in the Loop

Shapiro steps (Sliding state)



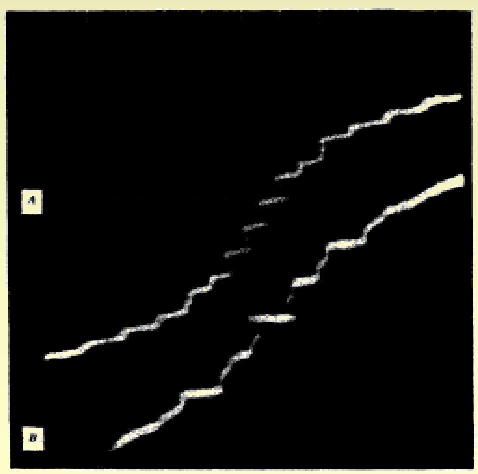
E>E$_T$ interference of phase

$$\frac{d^2\phi}{dt^2} + \Gamma\frac{d\phi}{dt} + \omega_0{}^2 \sin\phi = -\frac{ne\pi}{m^*}(E_{dc} + E_{ac}\sin\omega t)$$

$$\phi = at + b\sin(\omega t + \theta)$$

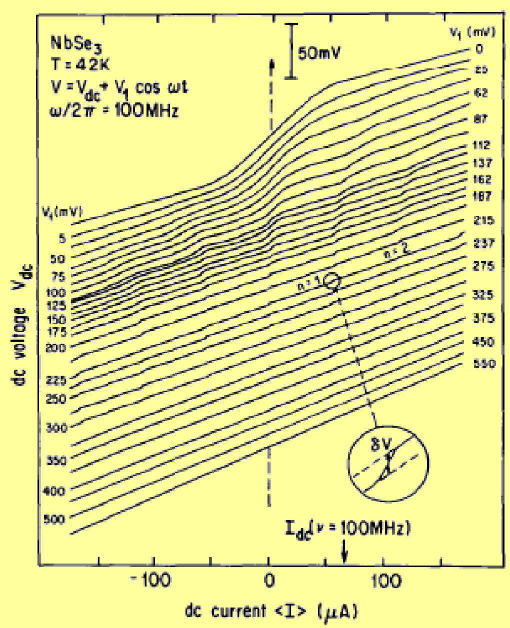
Fields

Josephson Junction Shapiro steps

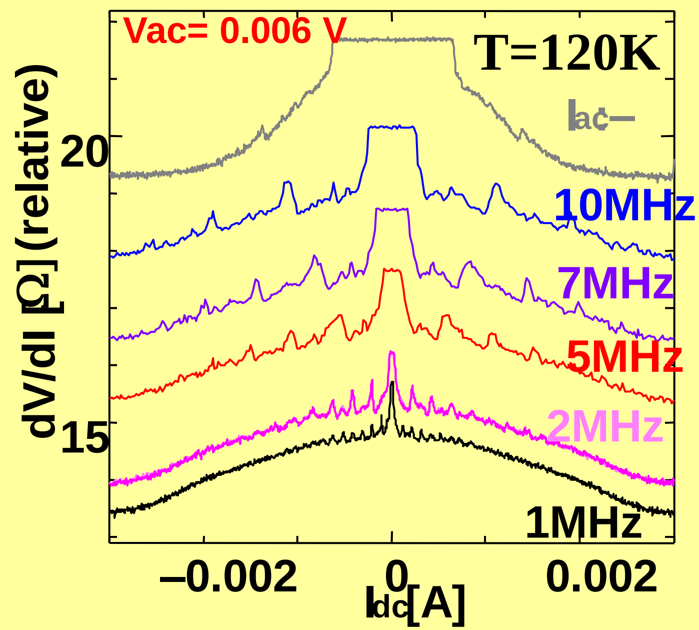


Shapiro, PRL, (1963)

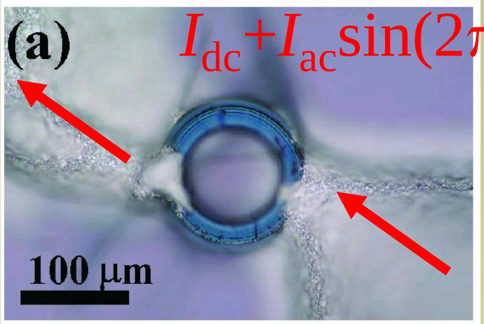
CDW:Shapiro steps : Cut-ring and needle NbSe



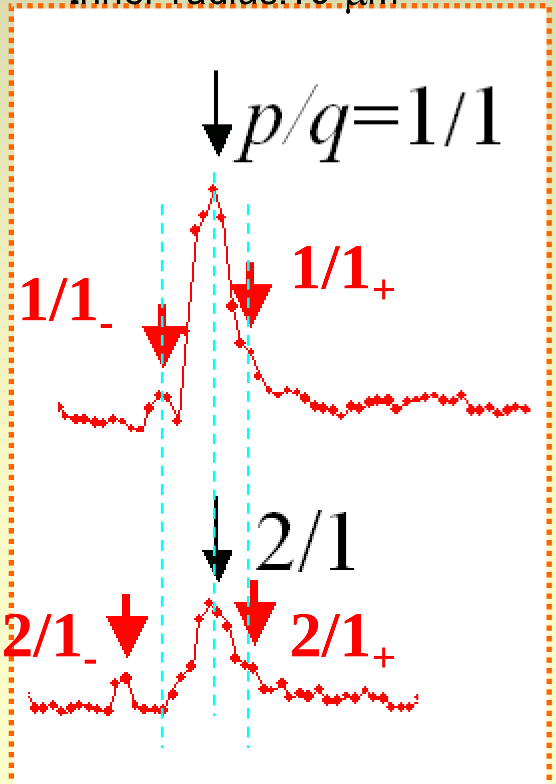
Zettel et.al, SSC, (1983)

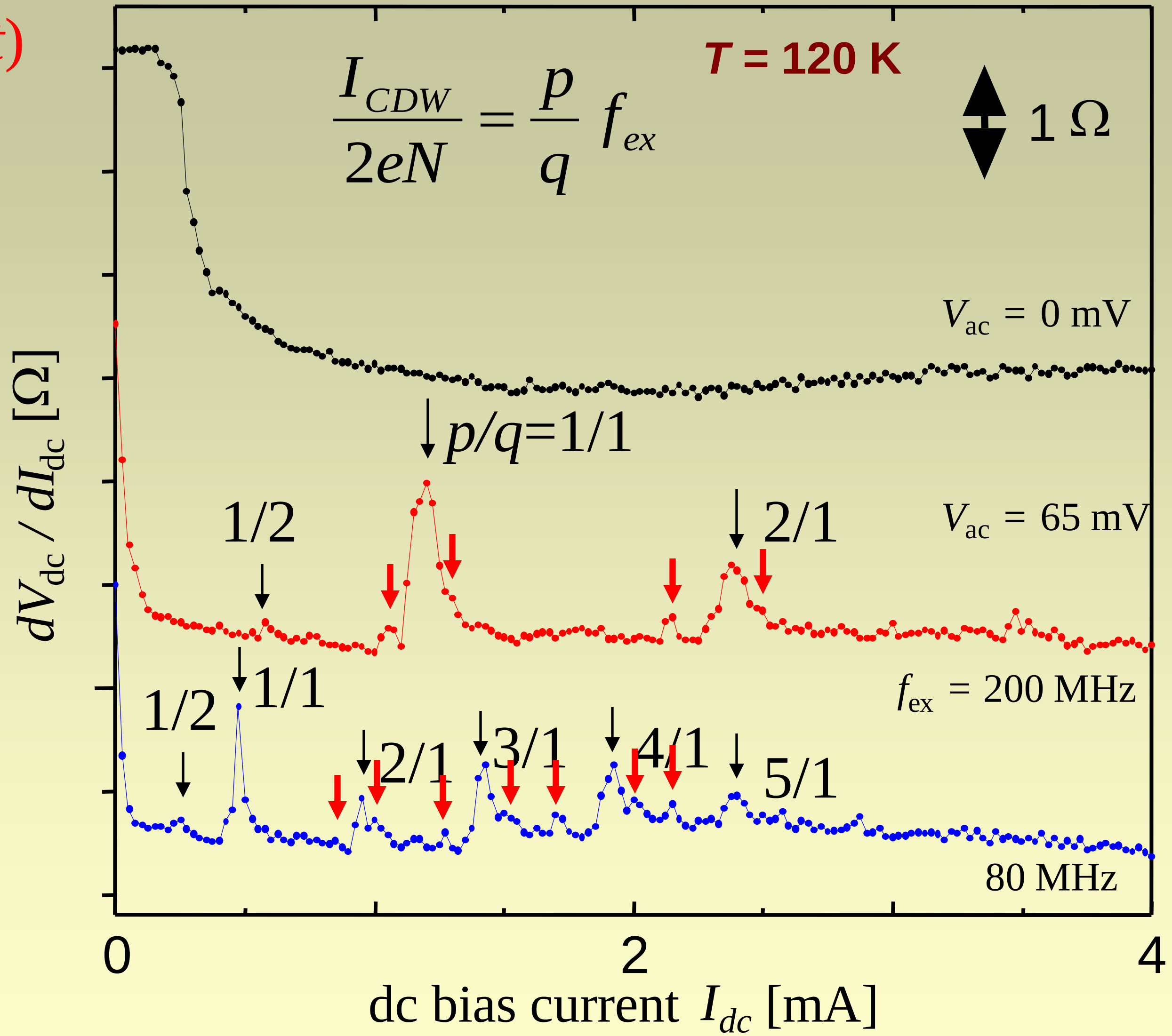


★★ Shapiro peaks in CDW loops



NbSe$_3$ ring
Outer radius : 120 μm
Inner radius:10 μm

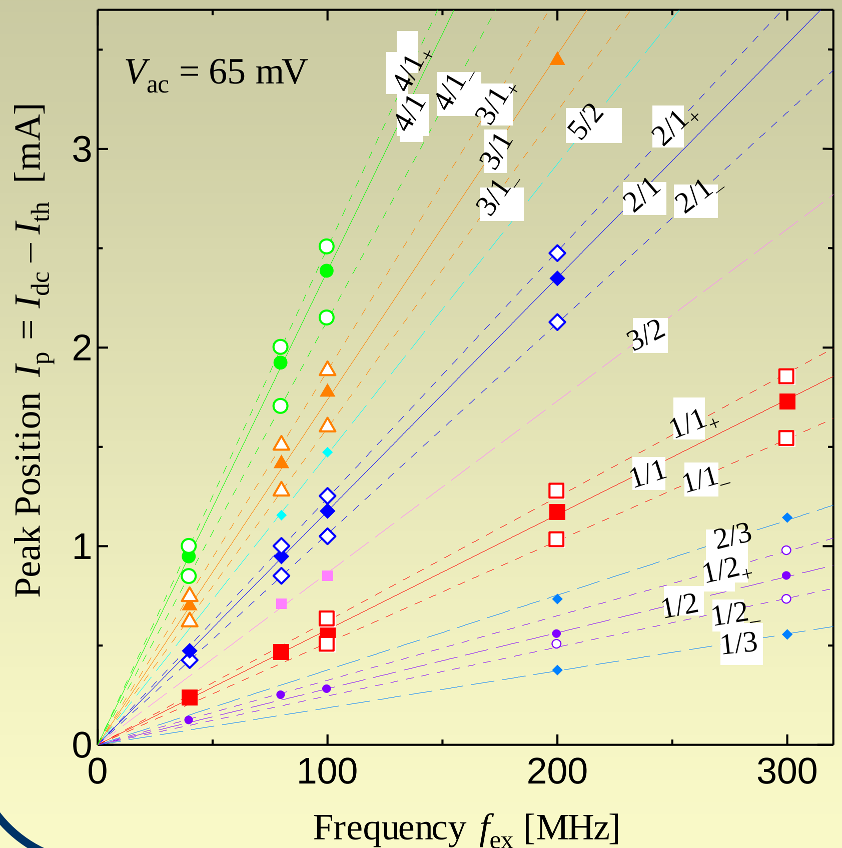




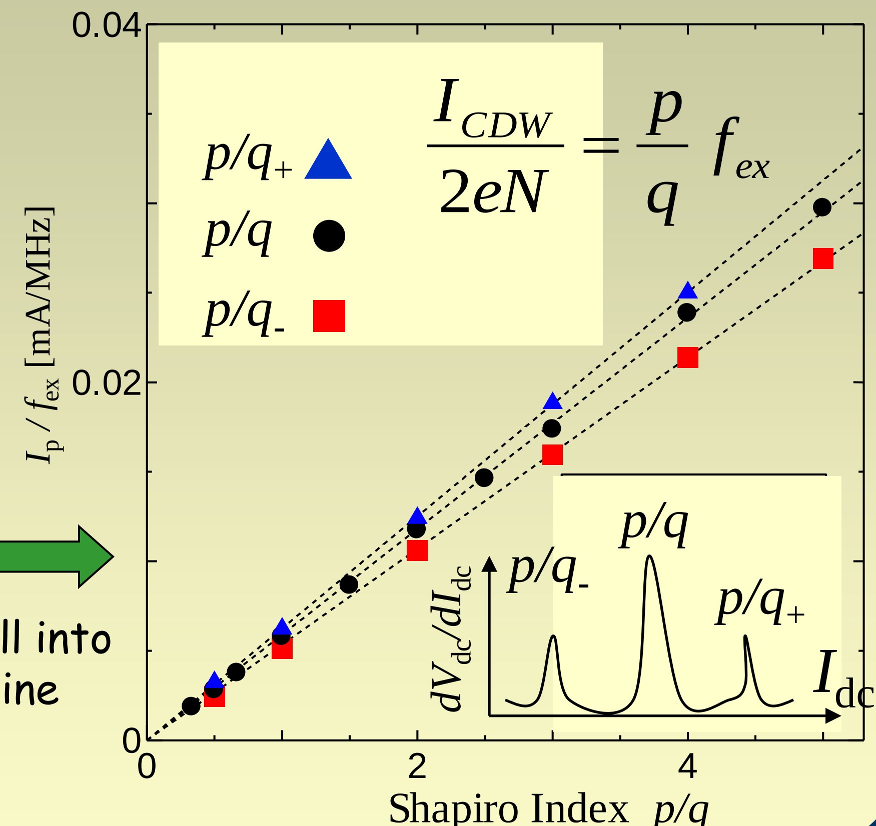
Subpeaks at both sides of Shapiro peaks are observed!

Analysis of peak positions

f_{ex} = 40, 80, 100, 200, 300 MHz

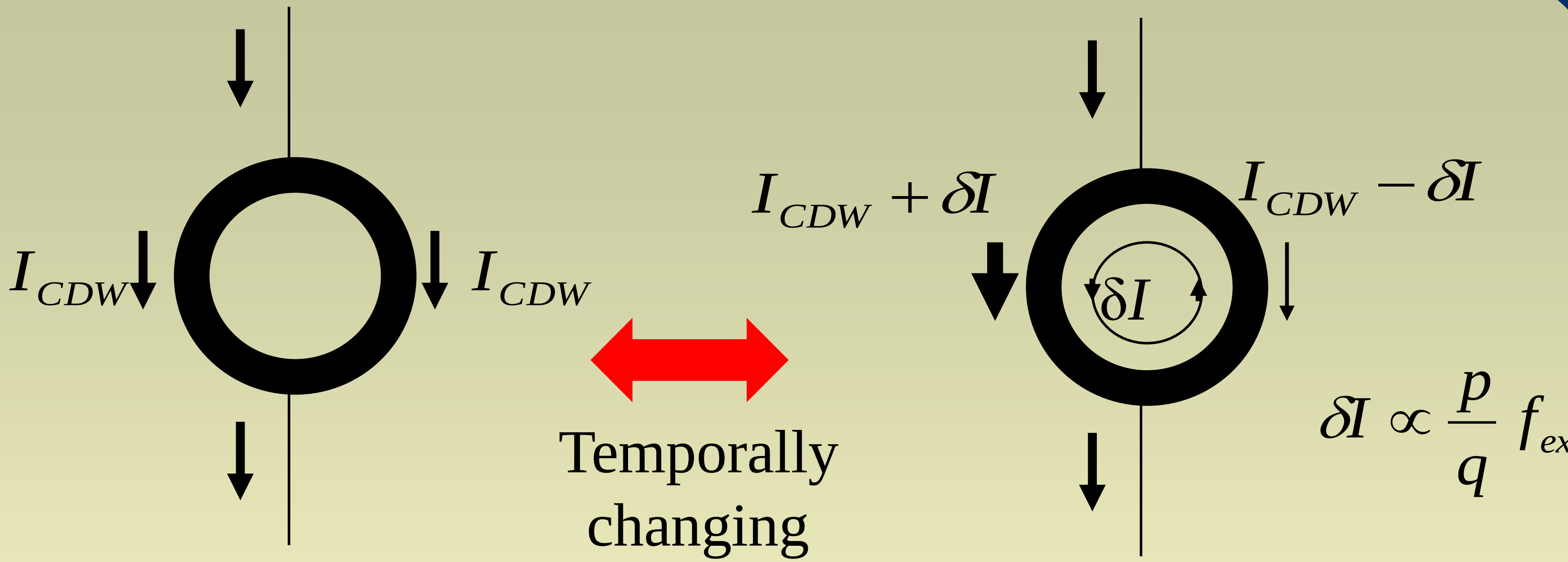


Fall into 3 line



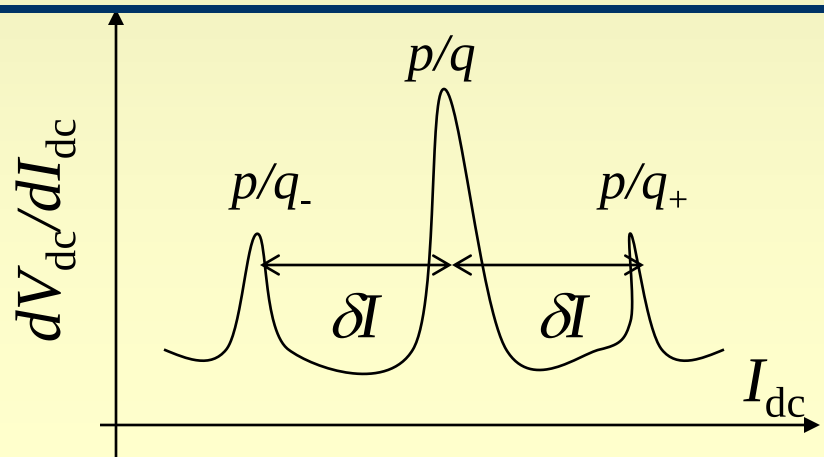
Discovery of Beat peak only in the loop CDW !!

Circulating current



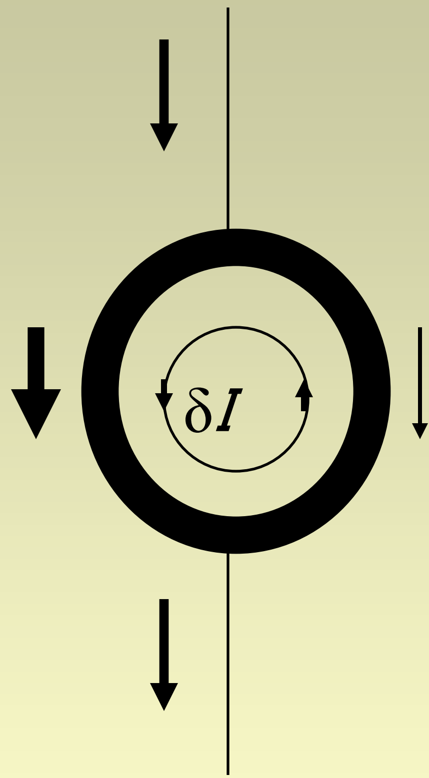
$$\frac{I_{CDW}}{2eN} = \frac{p}{q} f_{ex}$$

$$\frac{I_{CDW} \pm \delta I}{2eN} = \frac{p}{q} f_{ex}$$



Additional current must exist in the loop CDW

Damping time of Circulating current



If CDW is circulating, the damping time can be estimated by

Circumference: 10^{-4} m

Phason velocity: 10^{4} m/s

Circumference / Phason velocity = 10^{-8} s

>> $NbSe_3$ Phason damping time: 10^{-11} s

Richard and Chen, Solid State Commun. **86** 485 (1993)

Elongation of damping time !!

→ Precursor of Fröhlich supercurrent

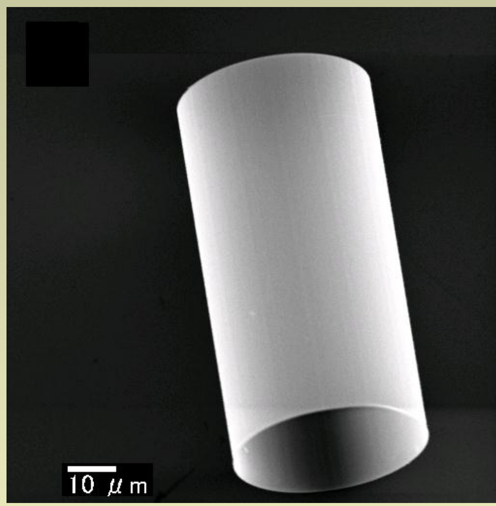
T. Matsuura, K. Inagaki, and S.Tanda, Phys.Rev.B 79, 014304 (2009).

Real space topology

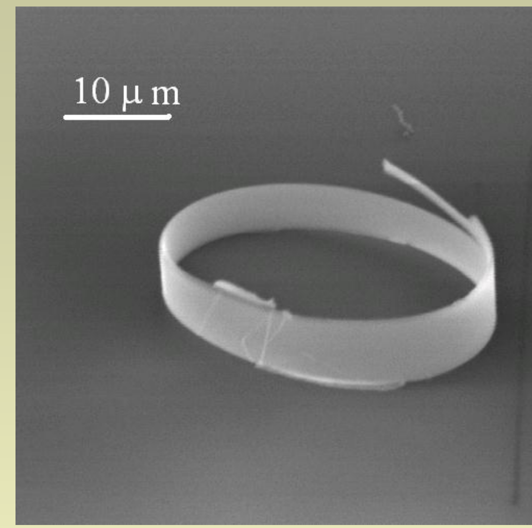
1. **Circulating current of CDW By Shapiro Steps in the Loop**

2. **AB Effect of CDW in the Loop**

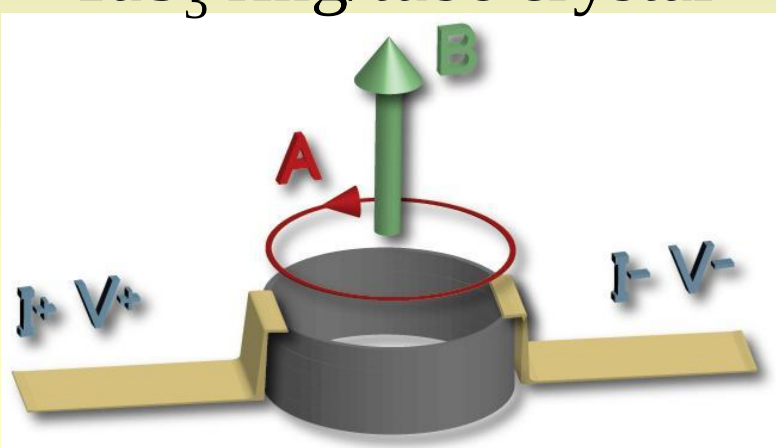
Experimental Setup

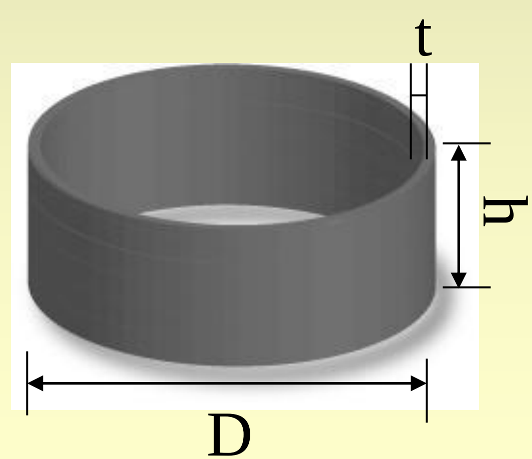


FIB cut



TaS_3 ring/tube crystal



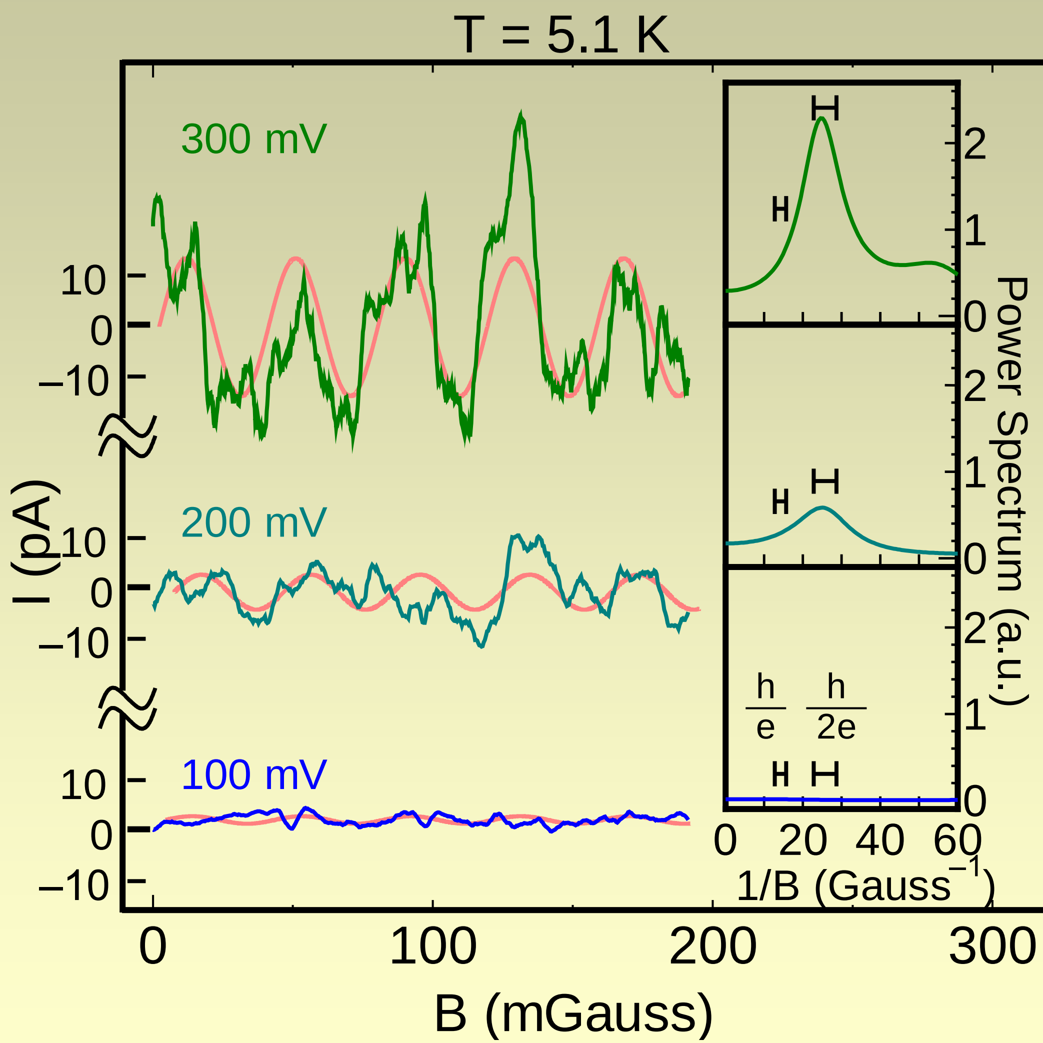


$t \approx 0.1 \mu m$

$h \approx 1 \mu m$

$D \approx 27 \mu m$

Result



Periodic oscillations were observed.

Estimation of unit charge

$$\Delta B = \frac{\Phi}{S} = \frac{h}{e^*} \cdot \frac{1}{\pi r^2}$$

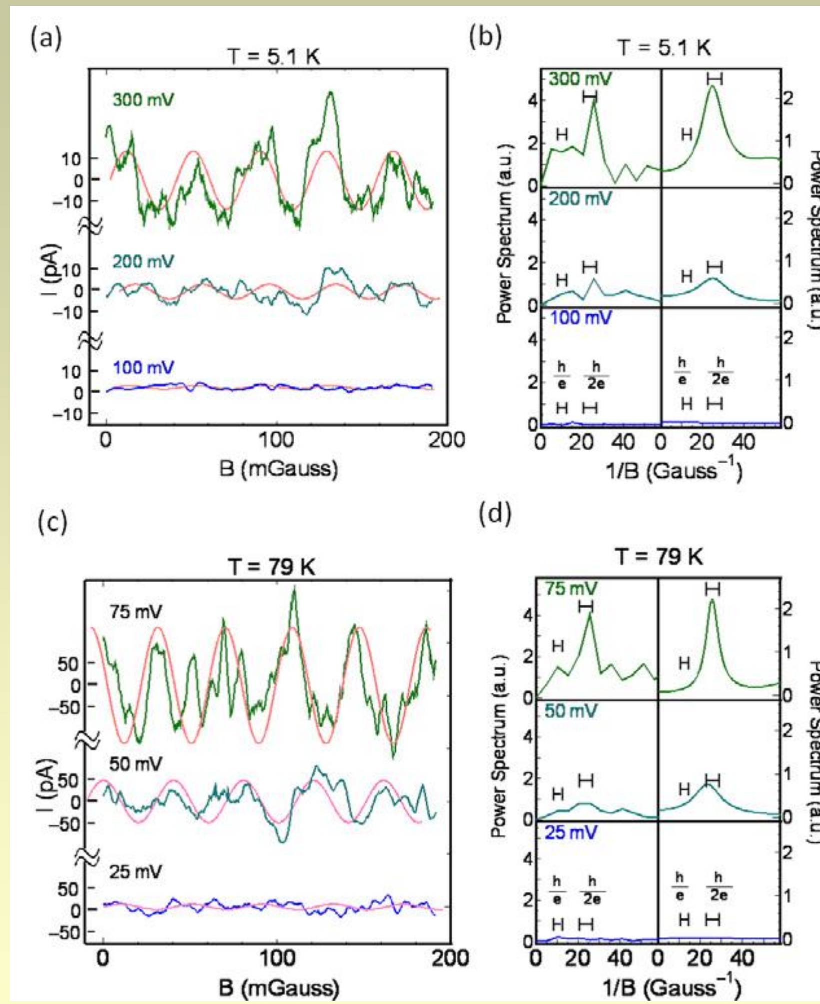
ΔB: The period of the oscillation

S: The area of the ring crystal

Sample	Diameter	Area m^2	Period	Charge
A	27 μm	5.6×10^{-10}	39.7 mGauss	3.0×10^{-19} C
B	17 μm	2.3×10^{-10}	95.2 mGauss	3.1×10^{-19} C

Unit charge corresponds to 2e (=3.2×10^{-19} C)

Quantum coherence at temperature of liquid-nitrogen



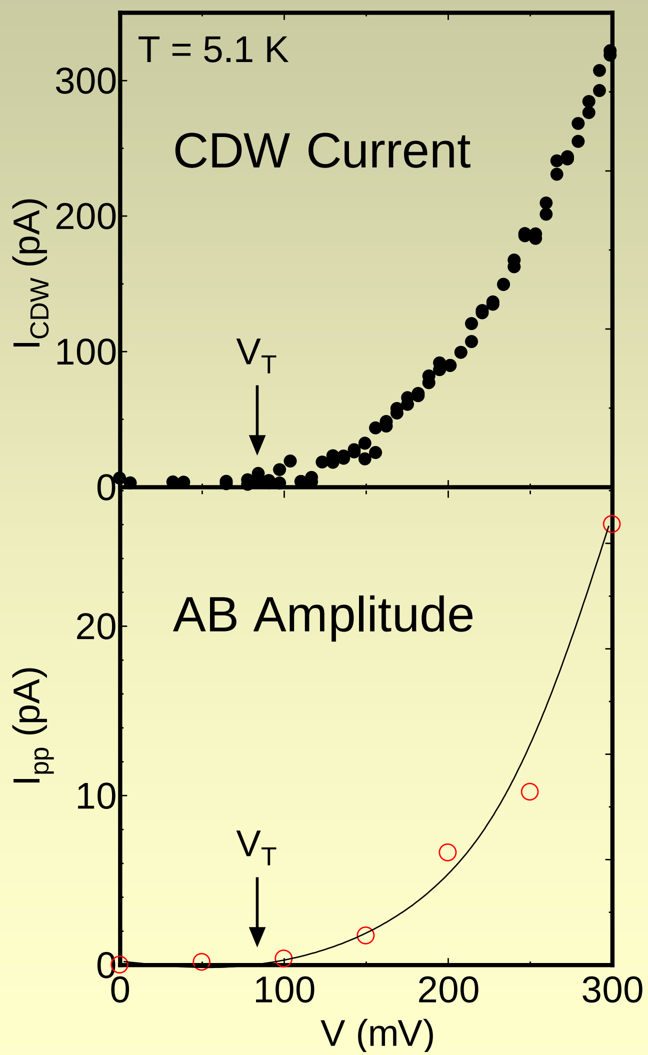
T=5.1K

T=79K

We also observed AB-effect of CDW sliding at 79K in TaS_3 ring crystals !

EPL, 97 (2012) 57011

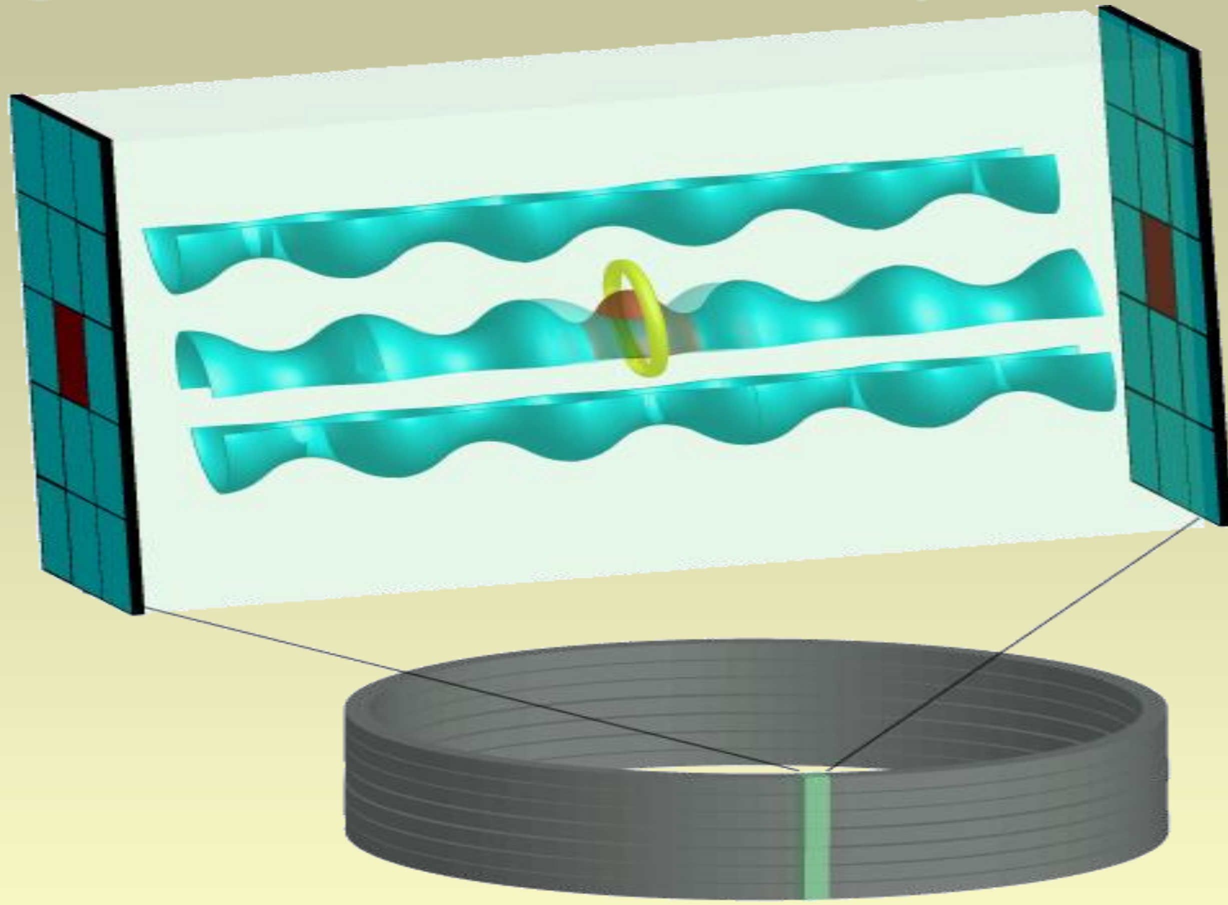
AB Amplitude



Two curves behave similarly

The amplitude of the interference is related with sliding CDW.

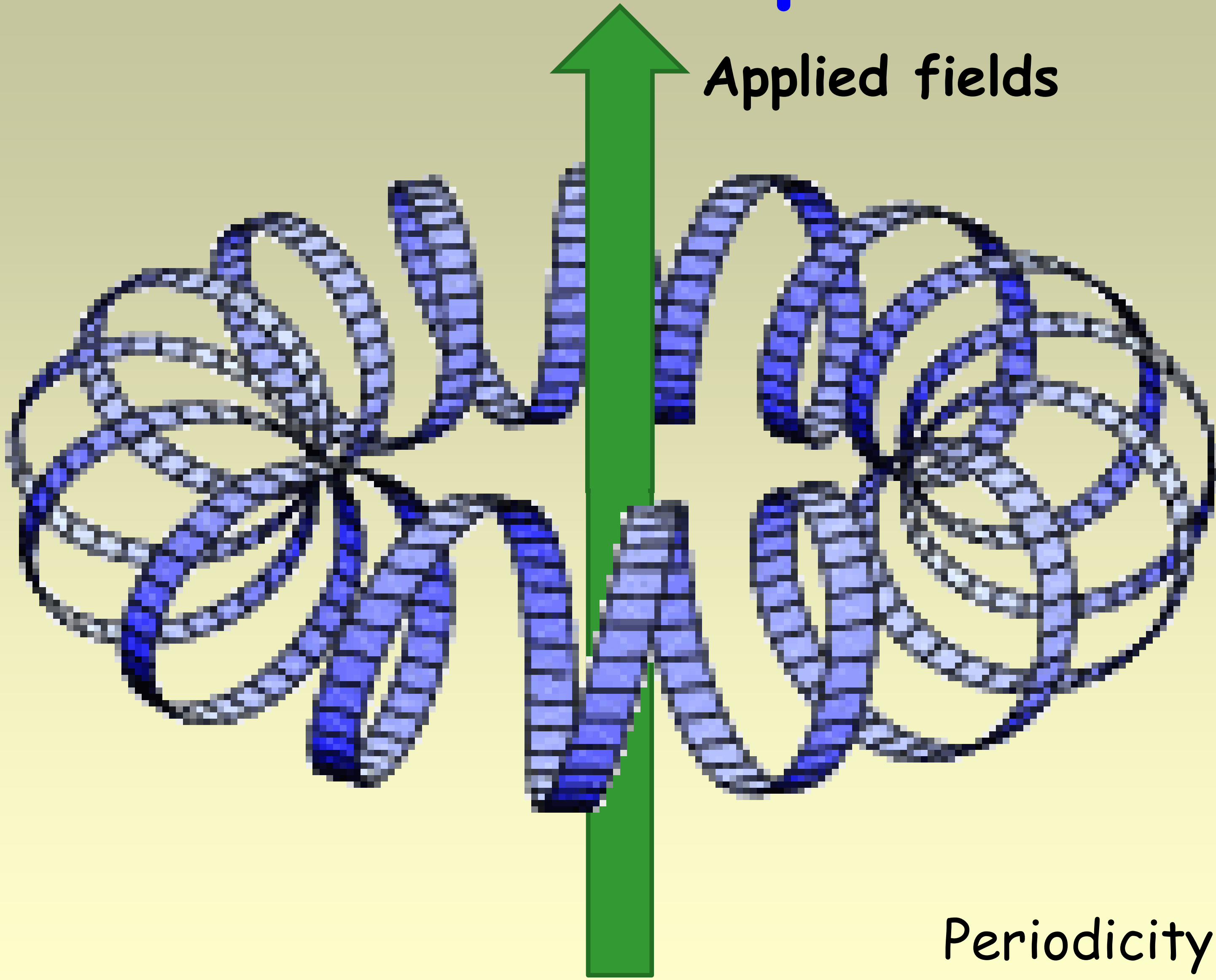
Edge dislocation loop : soliton



The dislocation loop of CDW cannot escape from a CDW chain, and provides 2e charge.

M. Tsubota, K. Inagaki, S. Tanda, Physica B: Condensed Matter Vol. 404, pp416-418, (2009), EPL (2012) in Press.

Quantum Phase Slip in CDW rings



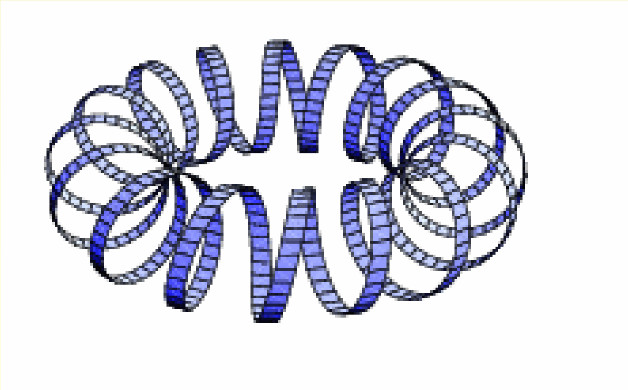
Periodicity $\frac{h}{2e}$

Periodicity $\frac{h}{2e}$

Sliding regime

$$\frac{AB\ amplitude}{CDW\ current} \approx 0.1$$

Ring CDW phase slips



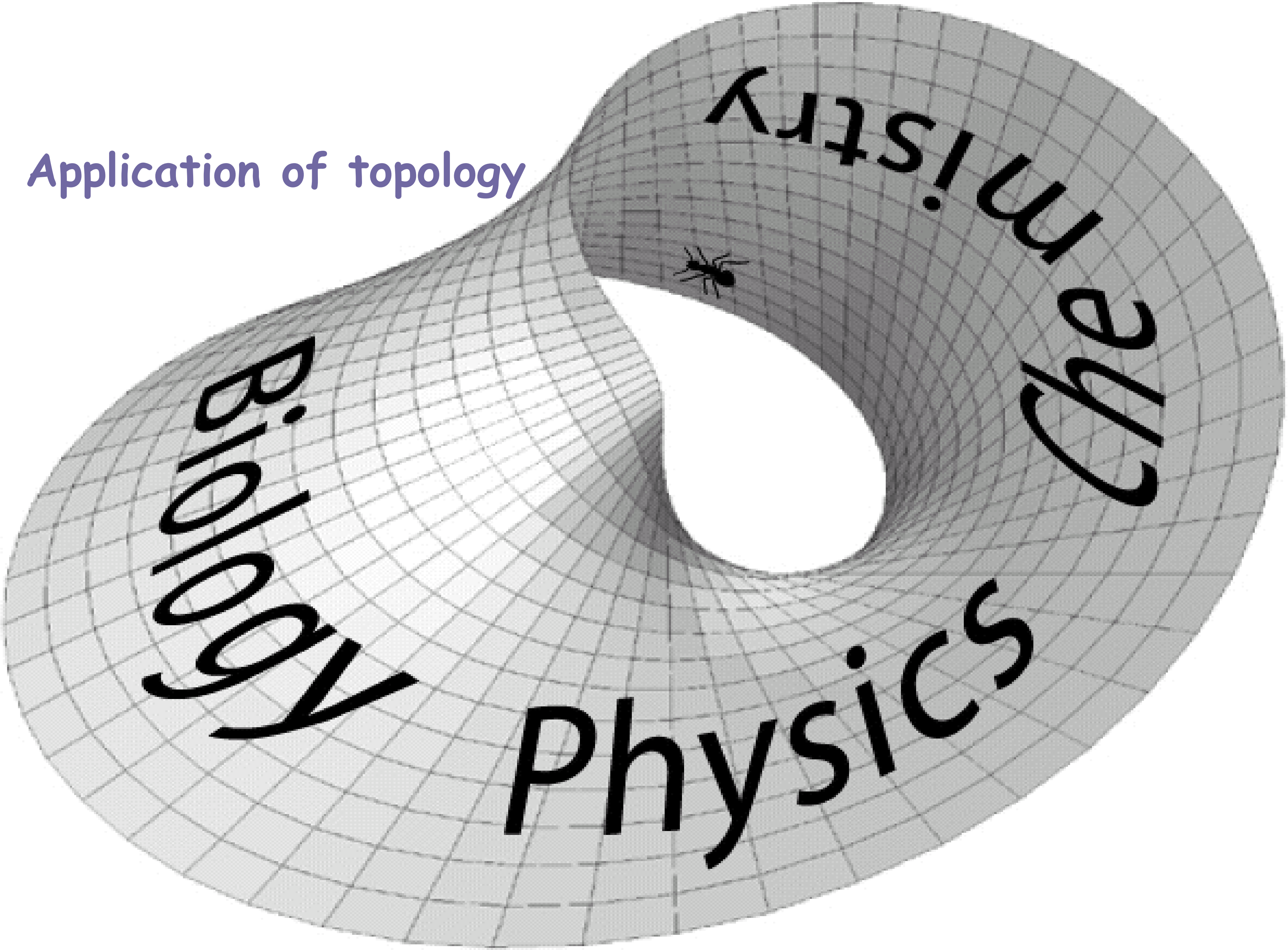
Application of topological crystals

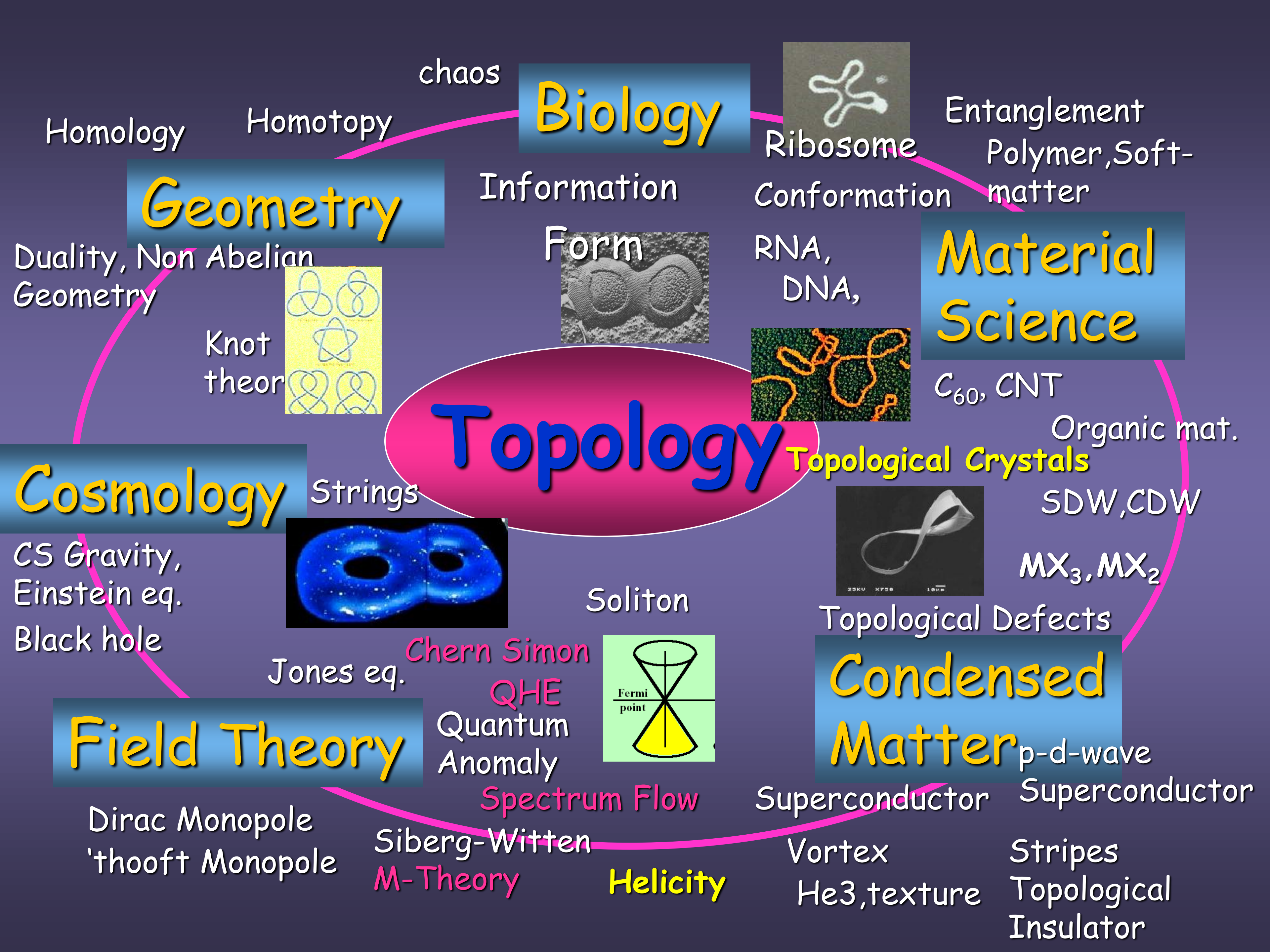
We obtain NbS_3 topological crystals  Tc=340K

, which have room-temperature Macroscopic-wave function (CDW) !

Room-temperature SQUID

Application of topology

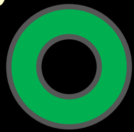




chaos
Biology
Homology
Homotopy
Ribosome
Entanglement
Polymer,Soft-matter
Geometry
Information
Conformation
Form
RNA,
DNA,
Material Science
Duality, Non Abelian Geometry
Knot theory
C60, CNT
Organic mat.
Topology
Topological Crystals
Cosmology
Strings
SDW,CDW
CS Gravity,
Einstein eq.
MX3,MX2
Soliton
Topological Defects
Black hole
Chern Simon
Jones eq.
QHE
Fermi point
Condensed Matter
Field Theory
Quantum Anomaly
p-d-wave Superconductor
Spectrum Flow
Superconductor
Dirac Monopole
'thooft Monopole
Siberg-Witten
M-Theory
Helicity
Vortex
He3,texture
Stripes
Topological Insulator

Topology change

Local Global effects

	Simply connected space : (Cut-ring) 	Multiply connected space :  
Crystals	Cycloid crystals (cut-ring) 	Ring, Mobius, 8, Hopf-link **Knots & links** crystals **Embedding** manifold
CDWs	Pinning of CDW at the edge	**Frölich superconductor** **AB effect** of CDW
Superconducting vortices	Abrikosov lattice	Cylindrical vortex in ring crystals Knots & Link vortex in 8 crystals

Topology defines the properties of condensed matter

Many Thanks !

色即是空

"Zen": Hakuin (Buddhist) used Mobius strip in 1758

色=空

色=*obverse*

空=*reverse*

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Prof. Inagaki Appl. Phys of H.U.
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